MAT116

Business Math: Part 1

Seattle Central Community College 2006–2007

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Table of Contents

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ACKNOWLEDGEMENTS

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THE CURRENT SET OF MATERIALS IS AN EVOLVING OUTGROWTH OF MEETINGS AND WORK BY **GERALD WRIGHT, JOHN TOUTONGHI, AND LAWRENCE MORALES**, ALL OF WHOM TEACH FOR SEATTLE CENTRAL COMMUNITY COLLEGE.

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Project 1:
Loan Foreclosure Decisions

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Chapter 1: Project 1–Introduction

Ch 1 Reading Questions

1. How are commercial loans repaid?
2. What do the phrases foreclose and work out mean?
3. What are the advantages and disadvantages of a loan workout?
4. What information will Acadia Bank use to decide whether to foreclose or workout?

Section 1-1: Business Background

A bank is a financial institution that transforms savings into capital investment. People and companies deposit excess cash in banks, which in turn lend these funds to other people and companies. The money is then invested in productive capital.

Banks make both personal and commercial loans. In a personal loan the borrower receives an amount of money which he or she pays back in monthly installments. The borrower also pays monthly interest on the outstanding principal. In a commercial or business loan the borrower keeps the entire amount of the loan for a fixed period of time, and then returns the principal as a lump sum payment. While he or she has the money the borrower must make regular payments of interest on the loan. For this project, we will only consider commercial loans.

The most important problem that a bank confronts is to whom it should lend. It makes this decision by processing information about the prospective borrower, and the proposed investments.

Since commercial loans are risky, the bank continually monitors borrowers. Should a borrower fail to make expected interest payments, the bank has to decide whether or not it should foreclose on the loan. In many cases, foreclosure means that the bank takes over ownership of a particular asset (the collateral for the loan), and sells this asset in the market.

If it does not foreclose, the bank can attempt to salvage the loan by setting up a new schedule of interest payments or by temporarily deferring such payments. These actions are called a loan work out.

The choice of whether to work out a loan or foreclose is a complex one. In many cases, foreclosure would force the borrower into bankruptcy. The key issue in a work out decision is the long-run viability of the borrower.

The bank has to decide whether the borrower has missed his or her recent payments because of temporary illiquidity or if there is a permanent problem. If it is the former, a work out may be indicated. If it is the latter, foreclosure may be appropriate.

Given the importance of the work out or foreclose decision, many banks that lend to smaller and medium size companies force the borrowers to have an exclusive borrowing relationship. This enables the lending bank to make the work out decision by itself.

To protect itself against litigation, a bank must base the decision about whether or not to attempt a work out arrangement on quantifiable considerations. This often involves the use of historical data on past borrowers. To be effective, the use of data must take into account specific information about an individual borrower and about the current economic conditions.

Section 1-2: Class Project

Acadia Bank has a commercial loan with a full value of $4,000,000 outstanding to John Sanders' entrepreneurial venture. This loan has a 10% interest rate, is one year old, and is scheduled to terminate in four years.

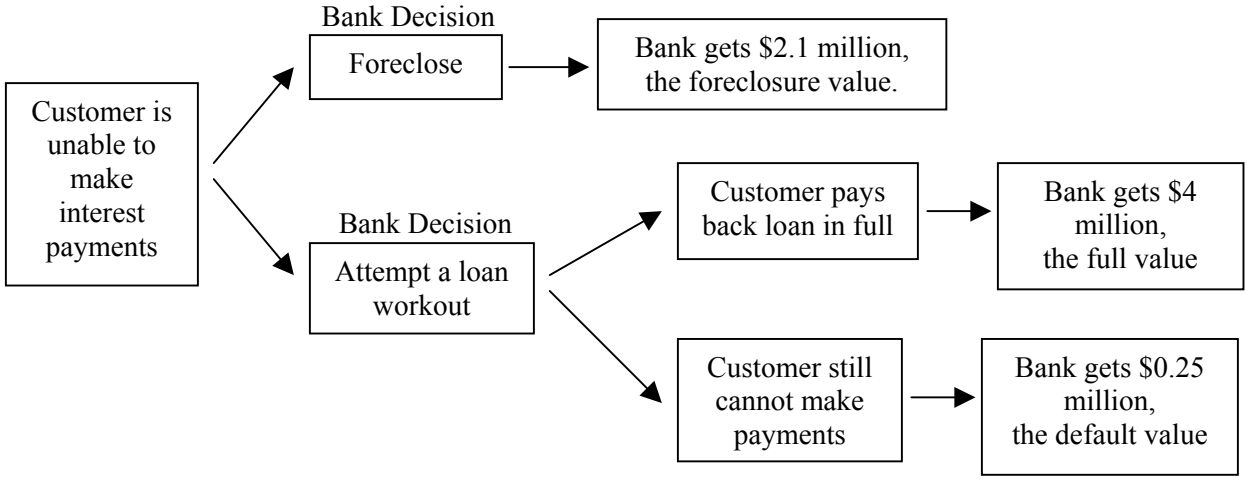
The last interest payment was due three weeks ago, and John was unable to make the payment. In fact, the bank has recently audited John's books, and discovered that, for the moment, the venture is illiquid, and will be unable to make interest payments for at least the next three months.

Acadia has to decide whether to foreclose on the loan, which would put John out of business, or to work out a new schedule of payments. Acadia knows that if it forecloses on John's loan, it will only recover a foreclosure value of $2,100,000 of the $4 million loan.

If it were to enter into a work out arrangement, there is some chance that the entire loan would be re-paid, and the on-going relationship with John would adequately compensate the bank for the costs of the work out and any lost interest.

If Acadia Bank attempts a work out, and John's venture fails anyway, then the business will have further decreased in value and the bank will only recover a default value of $250,000 on its loan.

The following diagram summarizes all the possible outcomes:



The problem confronting the bank is whether or not to work out the loan with John. On one hand, the bank can play it safe and foreclose. In this case they are guaranteed $2.1 million. On the other hand, the bank could take a risk and try a workout and "win big" by getting the entire $4 million. But a workout could also mean "losing big" by only getting 0.25 million. Is the chance of making $4 million worth the risk? What is the probability that John will pay back the loan?

Banks typically use data from previous clients to ascertain the probability of having a successful workout. Acadia Bank is the result of a merger between three banks. Unfortunately, each of these banks kept different kinds of records on the success or failure of past work out attempts.

The former BR Bank has work out records which include the number of years of experience that the entrepreneur had in his or her current line of business. The former Cajun Bank has work out records which include the entrepreneur's education status (whether the highest education level is: high school, a bachelor's degree, or a graduate degree). Finally, the former DuPont Bank has work out records which include the state of the economy at the time of the work out (whether the economy was in a recession, in normal times, or in a boom). Each of the three former banks made loans to very similar populations of borrowers.

Each of the former banks had data on several thousand individual work outs. The combined records that are available to Acadia Bank are shown in the Excel file ***Loan Records.xls.*** The first few rows of this spreadsheet looks like this:

Bank Information		**Borrower**			**Result**
Customer Number	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
1	Cajun		Bachelor's Degree		no
2	BR	11			no
3	BR	13			yes
4	Cajun		Graduate Degree		no
5	DuPont			Normal	no
6	BR	20			no
7	BR	1			yes

Conventional wisdom in the banking business indicates that the number of years of business experience of a borrower, educational level of a borrower, and the current economic conditions are all independent of each other. Moreover, these factors are independent, even if only successful loan work outs or only failed work out attempts are considered.

The following demographic and economic data characterize John's current situation.

John has seven years of experience in this kind of business.
John has a Bachelor's degree in Business Administration.
The economy is currently in "normal times."

Section 1-3: Focus on the Project

Each team is assigned one client who has defaulted on their loan. Using the data about your client, make an educated guess about whether the bank should foreclose the loan, or try a workout. At this stage, no calculations are required. The purpose of the preliminary report is to get your team thinking about the problem.

Study the Excel Spreadsheet ***Loan Records.xls***. On the menu, select **Data**, and use the **Sort** command to organize the information so you can more easily analyze the data related to your client.

Prepare a 5-minute presentation that will do the following:

- Introduce each person on the team.
- Introduce your client.
- Give background information about the client and the loan.
- State your conclusion (workout or foreclose).
- Give reasons for your conclusion.
- Discuss other possible information or tools you think you would need to make a better decision.

Chapter 2: Project 1–Set Theory

Ch 2 Reading Questions

1. Give an example of two sets, and draw a Venn Diagram which include some elements of the sets.
2. How are sets combined, and what notation is used?
3. Give an example of two disjoint sets.
4. What are some methods for counting the number of elements in a set?
5. What is double-counting, and how can it be avoided?
6. What do De Morgan's Laws tell us and how are they useful?
7. How is the Excel command DCOUNT related to set theory?
8. Read the Focus on the Project Section of the chapter and then briefly describe in your own words what we will do in this chapter to make help make progress on the team project.

Section 2-1: Introduction

In business, as well as many other fields, we are inundated with data. In order to make sense of it all, we need ways to organize and categorize this data.

In this project, we have a sizeable database of over 8,000 former clients. To help us organize this information, we could categorize the clients into groups. For instance, we can group together all the clients who defaulted on their loans during normal economic times. This aids in analyzing the information. Set theory is useful for organizing information such as this.

Definitions: Set, Element, and Set Theory

A **Set** is a group of things.

An **Element** is a member of a set.

Set Theory is the study of sets.

Notation

A set is usually represented by a capital letter, such as A or B.

Curly brackets { } indicate a set. Inside the curly brackets is either a description of the set, or a list of the elements in the set.

If a list of elements continues indefinitely, three dots … are used to indicate "and so on."

The symbol $\in$ means "is an element of."

The symbol $\notin$ means "not an element of."

Example 1:

Let R represent the set of all rich people. This set would include people like Oprah Winfrey, Bill Gates, and various wealthy CEO's of major corporations.

R = {all people whose individual net worth is greater than \$10 million}

Therefore, Oprah Winfrey $\in R$. ■

Example 2:

Let E be the set of all even whole numbers.

$E = \{0, 2, 4, 6, \ldots\}$

Therefore, $7 \notin E$. ■

Section 2-2: Combining Sets

Elements can often be members of more than one set. For example, Oprah Winfrey is rich as well as famous. Also, sets can be used to describe other sets. For instance, we could build the following sets:

{all people who are rich and all people who are famous}

and

{all people who are rich or people who are not famous}

Some notation was created to shorten the amount of writing needed for building sets.

Notation

^{C} (compliment) means "NOT"
For example, A^C means the set of all elements which are not in set A.

$\cup$ (union symbol) means "OR"
For example, $A \cup B$ means the set of all elements which are in set A *or* in set B.

$\cap$ (intersection symbol) means "AND"
For example, $A \cap B$ means the set of all elements which belong to set A *and* set B.

Example 1:

An employment agency is looking for workers who know how to operate Excel, and have at least one year of office experience.

Let E = {all people who know how to operate Excel}
Let Y = {all people with at least one year of office experience}

Describe the following in set notation:
a. E^C
b. $E \cap Y$
c. $E \cup Y$
d. $E \cap Y^C$

Solution:
a. E^C = {all people who do not know how to operate Excel}
b. $E \cap Y$ = {people who know how to operate Excel *and* have at least one year of office experience}
c. $E \cup Y$ = {people who know how to operate Excel *or* have at least one year of office experience}
d. $E \cap Y^C$ = {people who know how to operate Excel *and* do not have at least one year of experience}. ■

Example 2:

Alice, Ben, and Carl love to play team sports. The following sets give the sports each person likes to play.

U = {all team sports}
A = {baseball, soccer, hockey, football}
B = {soccer, football, lacrosse}
C = {hockey, basketball, rugby, baseball}

Describe the following:
a. $A \cup B$
b. $B^C \cap A$

c. $(A \cap C)^C$

Solution:

$A \cup B$ = {baseball, soccer, hockey, football, lacrosse}

$A \cap B$ = {soccer, football}

$B^C \cap A$ = {baseball, hockey}

$(A \cap C)^C$ = {all team sports except hockey and baseball}. ■

Section 2-3: More Definitions

Definitions: Empty Set and Universal Set

An **Empty Set** is a set with nothing in it. The notation is for an empty set is ∅.

A **Universal Set** is a set with everything in it. The notation is U, or sometimes S.

Example 1:

Let:

P = {all positive numbers}

N = {all negative numbers}

E = {all even integers}

Describe the following:

a. P^C

b. $P \cap N$

Solution:

a. P^C = {zero and all negative numbers}

b. $P \cap N = \varnothing$ (because no number can be both positive and negative). ■

Definition: Disjoint

Sets A and B called **disjoint** if they have no elements in common.

A more precise definition can be stated as follows:

Sets A and B are **disjoint** if and only if $A \cap B = \varnothing$.

In the above example, P (all positive numbers) and N (all negative numbers) are disjoint.

Example 2:

Let C = {cats} and let D = {dogs}.

C and D are disjoint because no animal is both a cat and a dog at the same time. Hence, $C \cap D = \varnothing$. ■

Definition: Subset

Set A is a **subset** of set B if and only if every element of set A is also an element of set B.

The notation for subset is $\subseteq$.

Example 3:

Let C = {cats} and let M = {mammals}.

C is a subset of M, because all cats are also mammals. This is written $C \subseteq M$. ■

Section 2-4: Venn Diagrams

Venn Diagrams are a useful tool for visualizing sets. Consider the following example:

Example 1:

Alice and Ben have been dating for a while, and they both love to go out to eat. However, they don't always agree on the restaurant. Alice likes Chinese, German, Italian, and Mexican food. Ben likes Italian, Mexican, Thai, Korean, and Ethiopian food. What are Alice and Ben to do?

Solution:
Let's organize this information using set theory:
Let A = {the types of restaurants Alice likes} = {Chinese, German, Italian, Mexican}
Let B = {the types of restaurants Ben likes} = {Italian, Mexican, Thai, Korean, Ethiopian}

Notice that they both like Italian and Mexican food. That's where their tastes in food intersect.

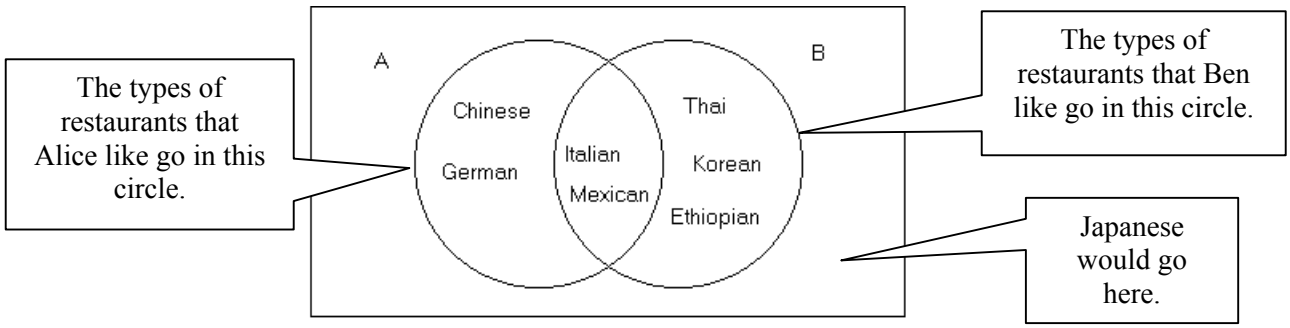
$A \cap B$ = {Italian, Mexican}

Answer: The solution to their dilemma would be go to an Italian or Mexican restaurant (or just stay home and have pizza!) ■

A diagram, called a Venn Diagram, can be made to represent the above information. Draw two overlapping circles. Circle A represents set A, and circle B represents set B. All the elements of set

A must fit inside circle A and all elements of set B must fit inside circle B. Elements that belong to both sets go in the overlapping area.

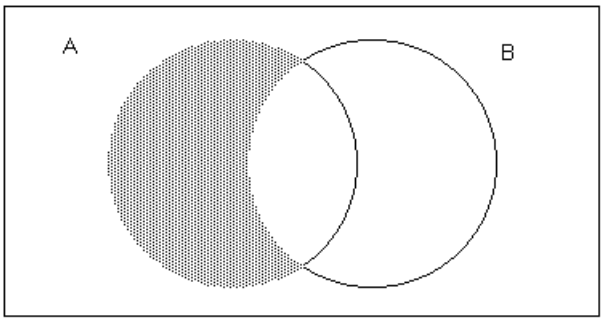
The Venn Diagram for Alice and Ben's situation would look like this:



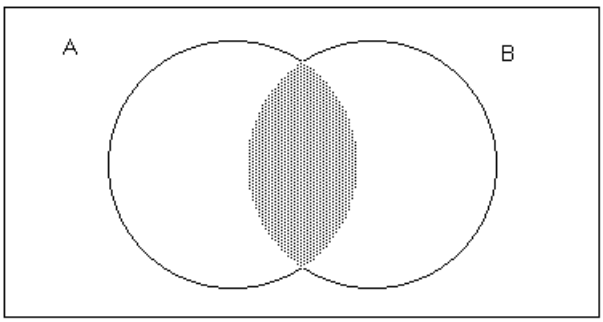
The rectangle represents the universal set. In this case, the universal set is all types of restaurants. Restaurants which neither Ben or Alice prefer would go outside the circles. For example, since neither of them appear to like Japanese food, that would be placed in the outer region.

Often it is not necessary to write every element inside the circles; just shading in an area is enough.

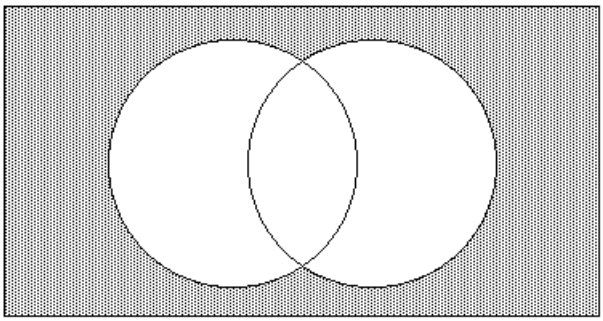
The shaded area of the following Venn Diagram represents all the restaurants which Alice likes but that Ben doesn't. (Or, more concisely, $A \cap B^C$).



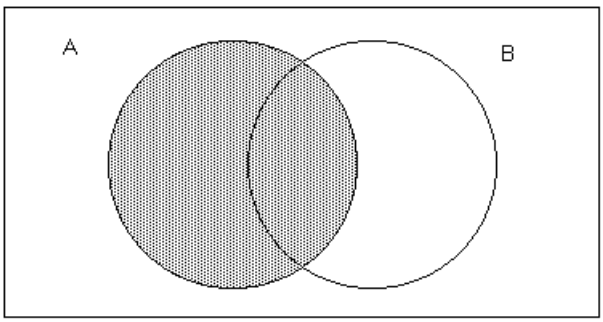
The following represents $A \cap B$:



The following represents $(A \cup B)^C$, which represents all restaurants that neither Alice nor Ben like.



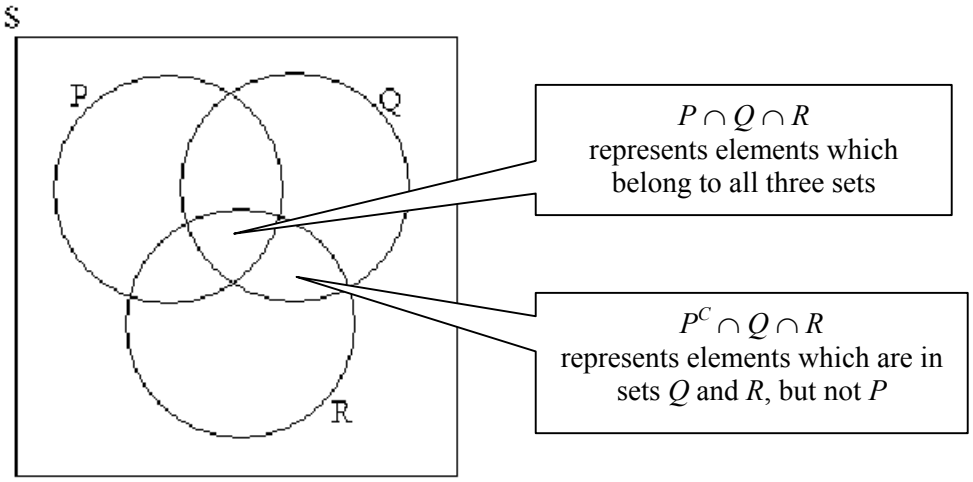
In the next example, the shaded area represents set A, and the white area represents A^C



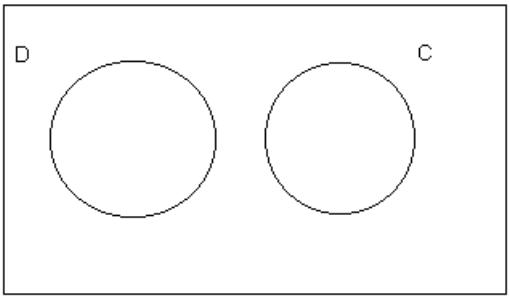
These are just a few examples of the many possible combinations using Venn Diagrams.

Section 2-5: Other Venn Diagrams

Venn Diagrams can be constructed for more than two sets. The main idea is to make sure the diagram is drawn in such a way that all the different possible overlaps are represented. The typical diagram for three sets looks like this:

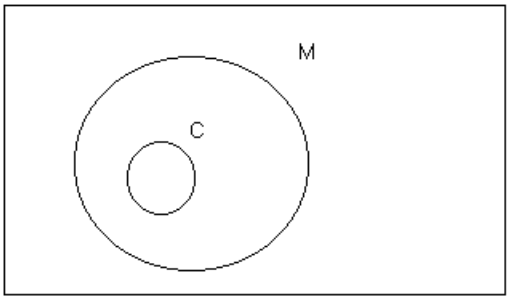


Also, Venn Diagrams can represent other situations as well. For instance, if two sets are disjoint (there is no overlap between the two sets), the Venn Diagram could be drawn like this:



D = {dogs} C = {cats}

If one set is a subset of another, this can be displayed as well:



C = {cats} M = {mammals}

As you can see, circle C is inside circle M, because all cats are also mammals.

Section 2-6: Counting the Number of Elements in a Set

Notation

n(A) means the number of elements in set A.

Example 1:

Consider Alice and Ben's situation once again:

A = {Chinese, German, Italian, Mexican}
B = {Italian, Mexican, Thai, Korean, Ethiopian}
$A \cap B$ = {Italian, Mexican}

n(A) = 4,
n(B) = 5
n($A \cap B$) = 2. ■

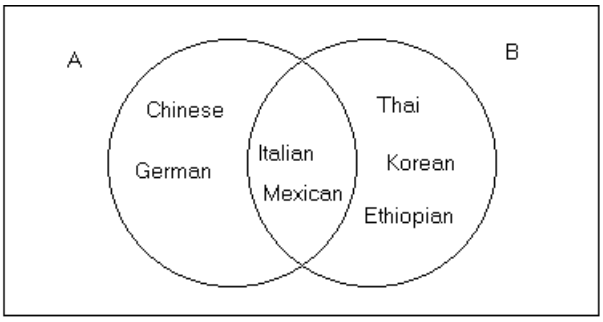
Simple addition and subtraction usually is sufficient to answer most counting questions.

Example 2:

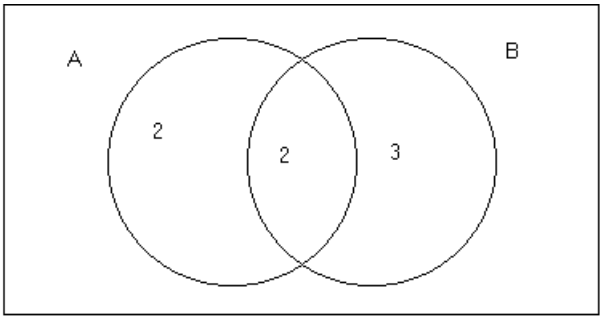
How many types of restaurants does Ben like that Alice does not like?
In other words, find n($A^C \cap B$).

Solution:
There are 5 foods which Ben likes, but 2 of those Alice also likes. So there are 3 foods which Ben likes which Alice doesn't. (5 – 2 = 3). ■

Recall the Venn Diagram for this situation:



If you only want the number of elements in each set, then draw this:



Notice that the numbers in circle *A* add up to 4, because there are 4 elements in set *A*. Likewise, the numbers in circle *B* add up to 5, because there are 5 elements in set *B*.

Example 3:

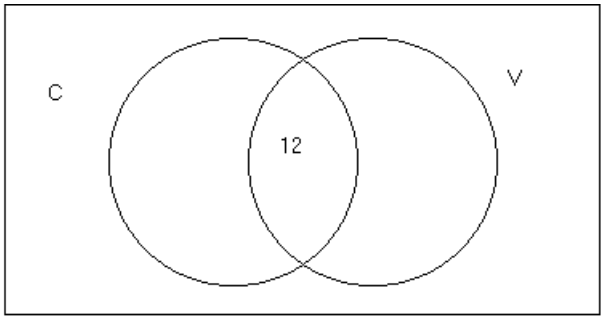
A researcher conducted a survey of 100 households. She found that 21 households have a computer, 56 have a VCR, and 12 have both a computer and VCR. Make a Venn Diagram and place the appropriate number in each of the 4 sections of the diagram.

Solution:
Let C = {all households in the survey which have a computer}.
Let V = {all households in the survey which have a VCR}.

$n(C) = 21, n(V) = 56, n(C \cap V) = 12$

Keep in mind that 21 is the *total* number of households which have computers. So *all* the numbers in the *C* circle must *add up* to 21. But just remember that some of them have VCR's and some do not. Likewise, all the numbers in circle *V* must add to 56.

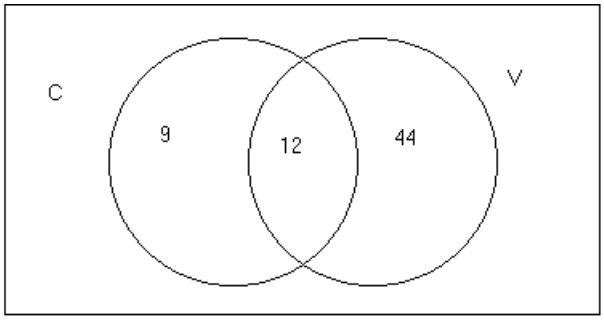
Start with the intersection $C \cap V$.



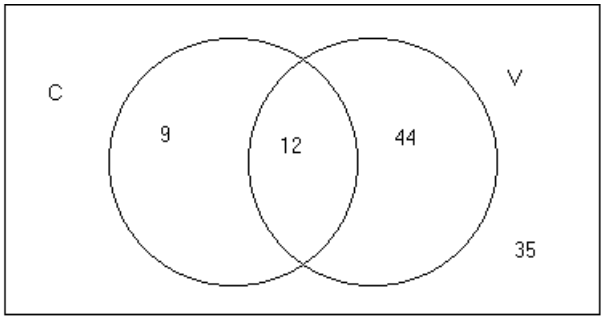
Now, we can figure out how many households have computers and don't have VCRs $\left(C \cap V^{C}\right)$:

$21 - 12 = 9$

We subtract because we know all the numbers in circle *C* must add to 21. Similarly, subtract 56 – 12 = 44 to find how many households have VCRs but not computers.



There is one more computation to do. Recall that there were 100 households in all. So far we have accounted for only 9 + 12 + 44 = 65. That means the remaining 35 do not have computers nor VCRs. Write this number outside both circles, but inside the rectangle.



This is the completed Venn diagram. ■

Section 2-7: Avoid the Double-Counting Error!

When counting the number of elements in a set, be sure not to count the same thing twice.

Example 1:

A researcher conducted a survey of 100 households. She found that 21 have a computer, 56 have a VCR, and 12 have both a computer and VCR. How many households have either a computer or VCR?

Solution:

Let's write this question symbolically:

$$n(C) = 21,\ n(V) = 56,\ n(C \cap V) = 12$$

Find $n(C \cup V)$.

It is tempting to just add 21 + 56 = 77. At first glance, this may seem reasonable: 21 households have computers and 56 have VCRs, so 77 must have either a computer or

VCR. But there is an error in this logic. There are 12 households which have *both* a VCR and a computer, so they are included in set C as well as set V. These households are counted twice in the final figure.

Correct Solution:

There are two ways to correctly compute the answer.

Method 1: Using the Venn Diagram above, add each section.

$9 + 12 + 44 = 65$.

Method 2: Add 21 + 56 together to get 77, then subtract 12 to compensate for the fact that these 12 households were counted twice.

$21 + 56 - 12 = 65$. ■

Here is a more formal statement of the idea we have been using…

General Counting Formula

If A and B are sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Subtracting compensates for the double-counting error.

Example 2:

A company operates two retail stores (Store A and Store B). They usually operate independently, but once per month they jointly operate a booth at a local street fair. In an attempt to inflate their monthly earnings reports, both stores included the revenue earned at the street fair. At the end of the month, Store A reported earning \$34,000 and Store B reported earning \$55,000. \$3,000 was earned at the street fair. What is the total revenue for the company from these two stores?

Solution:
If you consider set A to be all the dollars earned by Store A, and set B to be all the dollars earned by Store B, then

$n(A) = 34{,}000$
$n(B) = 55{,}000$
$n(A \cap B) = 3{,}000$

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 34{,}000 + 55{,}000 - 3{,}000 \\ &= \$86{,}000 \end{aligned}$$

Answer: The total revenue is \$86,000. ■

Example 3:

People at the marketing department researched the effectiveness of their advertising. They sent the following survey to their customers:

> **Survey**
> How did you hear about our company?
> (check all that apply)
>
> ___ Television ___ Radio

A total of 450 customers returned the survey with one or both choices marked. 300 of the respondents checked "Television", and 200 checked "Radio". How many checked both?

Solution:
Let T = {those who heard of the company through an ad on television}
Let R = {those who heard of the company through an ad on the radio}

We know that $n(T) = 300$, $n(R) = 200$, and $n(T \cup R) = 450$.

We need to find $n(T \cap R)$.

The Counting Formula works for this situation. Let $x = n(T \cap R)$

$$\begin{aligned} n(T \cup R) &= n(T) + n(R) - n(T \cap R) \\ 450 &= 300 + 200 - x \\ 450 &= 500 - x \\ -50 &= -x \\ 50 &= x \\ x &= 50 \end{aligned}$$

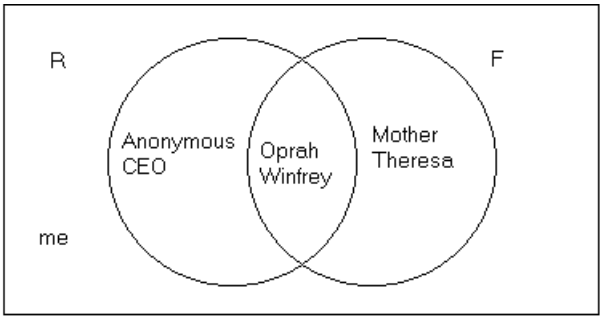
Answer: Therefore, 50 people heard about the company through both television and radio ads. ■

Section 2-8: DeMorgan's Laws

In 1847, an English mathematician and logician, Augustus de Morgan (1806–1871) proposed two useful facts about sets which are now named after him. We will explore them here, but only briefly.

Consider the group of rich and/or famous people. Some, like a CEO of a large corporation, are rich but not famous. Some, like Mother Theresa, are famous but not rich. Some, like Oprah

Winfrey, are both. Unfortunately, I am neither rich nor famous. A Venn Diagram of this situation would look like this:



There are two ways to describe those unlucky folks like myself which are outside both circles:

$(R \cup F)^C$ = those who are not in the group of rich OR famous people.

or

$R^C \cap F^C$ = those who are not rich AND are not famous.

So, these two expressions are equal. In other words if you're not in the group of rich or famous people, then this means you are not rich and you are not famous.

This brings us to the first of DeMorgan's Laws:

$$(R \cup F)^C = R^C \cap F^C$$

The second law is similar. Consider those lucky few, like Oprah Winfrey, that are both rich and famous. This comprises the inner part of the Venn Diagram. Now consider everyone else: those who are not in the group of rich and famous people, or

$$(R \cap F)^C$$

If you are not in the rich and famous group, then this means either you are not rich, *or* you are not famous. This can be written:

$$R^C \cup F^C$$

Therefore,

$$(R \cap F)^C = R^C \cup F^C$$

To really understand what these laws tell you, you might want to start with blank diagrams and try to shade the picture so it portrays the left side of the equation. Then use another blank diagram to shade the right side. When you are done, the two shadings should match.

The two rules are summarized below.

DeMorgan's Laws

If A and B are sets, then:

$$(A \cup B)^C = A^C \cap B^C$$

and

$$(A \cap B)^C = A^C \cup B^C$$

The laws are a useful tool for counting the number of items in a set.

Example 1:

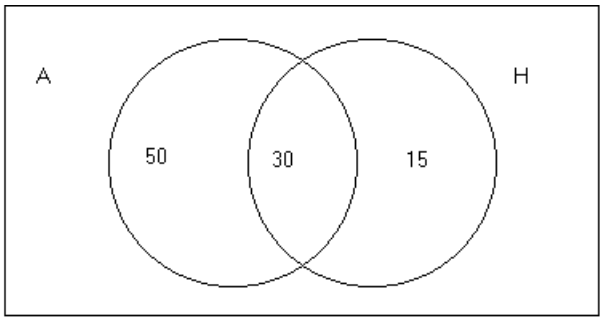
Suppose a travel agency has records for their past 100 clients. Of these clients, 80 have purchased an airplane ticket, 45 have booked a hotel room, and 30 both purchased a ticket and booked a hotel room. How many have not purchased an airplane ticket nor have not booked a hotel room?

Let A = the set of clients who purchased an airplane ticket.
Let H = the set of clients who booked a hotel room.

Find $n(A^C \cap H^C)$.

Solution (Method 1):

Set up a Venn Diagram. Enter 30 for $A \cap H$, then subtract to get the other two sections.



By DeMorgan's Law, we know that the clients who didn't buy an airplane ticket and also didn't book a hotel room, $A^C \cap H^C$, is the same as $(A \cup H)^C$, which is the area outside the circles of the Venn Diagram. Since there are 50 + 30 + 15 = 95 clients accounted for inside the circle, and there are 100 clients altogether, then there must be 5 clients outside the circles.

So, $n(A^C \cap H^C) = 5$.

Answer: There are 5 clients who have not purchased an airplane ticket nor have not booked a hotel room. ■

Solution (Method 2):

We can also use the general counting rule:

$$\begin{aligned} n(A \cup H) &= n(A) + n(H) - n(A \cap H) \\ &= 80 + 45 - 30 \\ &= 95 \end{aligned}$$

Since there are 100 clients in all, then $n(A \cup H)^C = 100 - 95 = 5$.

Since $A^C \cap H^C$ is the same as $(A \cup H)^C$, then $n(A^C \cap H^C) = 5$ as well.

Answer: There are 5 clients who have not purchased an airplane ticket nor have not booked a hotel room. ■

Section 2-9: DCOUNT and Set Theory

You will need the Excel file ***cars.xls*** for this section. Also, you should already be familiar with the Excel function DCOUNT before beginning this section.

When there is a large database, Excel can be used to count the number of elements in different sets.

The ***cars.xls*** file is a record of vehicles involved in low speed collisions. The first few rows look like this:

Model	Model Year	Location of Repair	Estimated Speed (mph)	Repair Cost
Jeep	1994	Utah	09	$1,000
Nissan	1995	Arizona	06	$666
Ford	1994	Colorado	10	$2,129
Jeep	1996	New Mexico	13	$2,666

Example 1:

Let J = {all Jeeps in the database}. Let A = {all cars in the database which were repaired in Arizona}.

a. Find $n(J \cap A)$.
b. Find $n(J \cup A)$.

Solution a:
This first question is asking for the number of vehicles which are Jeeps *and* which were repaired in Arizona.

Set up the criteria but make sure it's all on one line so that Excel will find rows in the database that contain both the words Jeep in the Model column and the word Arizona in the Location of Repair column:

Model	Model Year	Location of Repair	Estimated Speed (mph)	Repair Cost
Jeep		Arizona		

Let's assume this criteria was put in cells H12:L13. Also, the original database is in cells B12:F384.

Then use the DCOUNT command, with the following arguments:

Database: B12:F384
Field: (leave blank)
Criteria: H12:L13

Answer a: 38. ■

Solution b:
Here, we are asked to find the number of vehicles that are Jeeps or were fixed in Arizona.

Because we are using logical operator OR, we need two rows in the criteria:

Model	Model Year	Location of Repair	Estimated Speed (mph)	Repair Cost
Jeep				
		Arizona		

Use the DCOUNT command with the following arguments:

Database: B12:F384
Field: (leave blank)
Criteria: H12:L14

Answer b : 178. ■

There are 178 rows in the database which contain the word Jeep in the "Model" column OR the word Arizona in the "Location of Repair" column. You might be wondering if there are some cars that were Jeeps AND were repaired in Arizona. In other words, did Excel double-count these? The answer is no! DCOUNT is programmed to avoid the double-counting error.

In fact, if you want to check for accuracy, you could count the number of Jeeps. $n(J) = 105$. Also count the number of vehicles repaired in Arizona: $n(A) = 111$.

We know from part (a) that $n(J \cap A) = 38$.

Now we can use the general counting formula:

$$n(J \cup A) = n(J) + n(A) - n(J \cap A) = 105 + 111 - 38 = 178.$$

Section 2–10: Focus on the Project

How can set theory help us with the decision on whether or not to attempt a loan work out?

Open the Excel file ***Loan Records.xls,*** and you will find a database of over 8,000 former clients. In our example, the client has 7 year's of experience, a Bachelor's Degree, and is operating in normal economic times. Let's see how many clients in the past met these same criteria, and how many actually paid back the loan.

S = The set of all former workout clients who successfully paid back their loan.
F = The set of all former workout clients who failed to pay back the loan.
Y = The set of all former workout clients who have 7 years of experience.
T = The set of all former workout clients who have a Bachelor's Degree. Think of T for "training."
C = The set of all former workout clients who operated during normal economic times. Think of C for "condition" of the economy.

Suppose we want to find $n(Y \cap S)$. $n(Y \cap S)$ means we want to find the number of former workout clients who had 7 years of experience AND successfully paid back their loan. Keep in mind that only BR Bank kept records on years of experience. We will use DCOUNT will to find the answer.
DCOUNT can be used to extract data from the Acadia Bank records (***Loan Records.xls***) that will help us use the particular characteristics of the John Sanders' loan in making a work out decision. In particular, we can gather data on the success of work outs, and on the borrowers from the individual banks that merged to form Acadia Bank.
For example, let Y be the set of borrowers that have 7 years of experience in the business. Let T be the set of borrowers has a Bachelor's Degree. Let C be the set of borrowers who defaulted when the economic conditions were normal.
Let the subscript BR represent BR bank. So, S_{BR} means the set of successful workouts at BR bank, and F_{BR} is the set of failed workouts at BR bank. Likewise, let the subscript DP represent Dupont Bank, and CJ represent Cajun bank.

Example 1:

Solution:
$n(Y \cap S_{BR})$ is the number of clients in the database that have 7 years experience AND had a successful loan workout through BR bank.
First, we set up the DCOUNT criteria:

	I	J	K	L	M	N
10	**Customer Number**	**Former Bank**	**Years In Business**	**Education Level**	**State Of Economy**	**Loan Paid Back?**
11		BR	7			yes

Then, select DCOUNT from function wizard (the f_x symbol). The arguments would be:

Database: B10:G8236
Field: (leave blank)

Criteria: I10:N11

The result is 105. This means there were 105 former clients who had 7 years of experience and paid back the loan.

Answer: $n(Y \cap S_{BR}) = 105$. ■

You are now read to go to the section titled ***Project Specifics*** and do ***Part 1.***

Chapter 2 Practice Problems

For the following problems, let A = {1, 2, 3, 4, 5, 6}, let B = {a, b, c, d, e}, and let C = {all even integers} Write true or false for each statement. If the statement is false, write a true statement for it.

1) $4 \in A$
2) $t \in B$
3) Tom $\notin$ {Joe, Frank, Jim}
4) $6 \notin C$
5) $A \subseteq$ {all positive numbers}
6) $B \subseteq$ {all letters of the alphabet}
7) $A \cap B = \{1, 2, 3, a, b, c\}$
8) $A \cap C = \{2, 4, 6\}$
9) $B \cup \{f, g, ..., z\}$ = {all letters of the alphabet}
10) $A \cup B = \{1, 2, 3, a, b, c\}$
11) A and C are disjoint
12) B and C are disjoint

For the following problems: Alice, Ben, and Carl love to play team sports. The following sets give the sports each person likes to play.

U = all team sports

A = {baseball, soccer, hockey, football}

B = {soccer, football, lacrosse}

C = {hockey, basketball, rugby, baseball}

Find the following :

13) $A \cup C$
14) $B \cap A$
15) $B \cap A^C$
16) $B \cap C$
17) Describe $(B \cup C)^C$ in words

For the following problems, let the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
Let X = {1, 2, 3}, let Y = {2, 4, 6, 8, 10}, and let Z = {4, 5, 6}. List the members of the following sets:

18) $X \cap Y$
19) $X \cup Y$
20) X^C
21) $X \cap Z$
22) $X^C \cup Y$
23) $Z^C \cap Y^C$

24) Alice and Ben like to go to movies together, but they don't always agree on what movie to see. Alice likes romantic comedies, documentaries, and anything with Jackie Chan in it. Ben likes romantic comedies, war movies, and anything with Jim Carrey. Make a Venn Diagram of this situation.

25) Business A sells office supplies, computers, and radios. Business B sells office supplies, cell phones, and computers. Make a Venn Diagram of this situation.

26) I had a party and invited all my rich friends and all my famous friends. There were 18 rich friends, and 12 famous friends. Out of these people, 5 of them were rich *and* famous. Draw a Venn Diagram and place the appropriate number in each section. How many people were at my party?

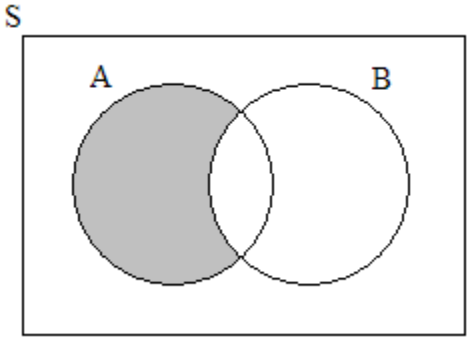
27) A survey of 300 pet owners, 120 had a cat, 150 had a dog, and 45 had a cat and a dog. Draw a Venn Diagram and place the appropriate numbers in each section. How many had neither a cat nor a dog?

28) At my next party, I had 20 guests (all were rich and/or famous). 3 of them were rich *and* famous, 12 were famous but not rich. Draw a Venn Diagram and place the appropriate number in each section. How many rich but not famous?

29) Out of 50 people, 30 of them spoke Spanish, 25 of the Spanish speakers did not speak French. 10 spoke neither Spanish nor French. Draw a Venn Diagram and place the appropriate number in each section. How many spoke French?

For the following problems, find an expression using the $\cup$, $\cap$ and ^{C} (Compliment) notation that is equivalent to the shaded areas shown. (There may be more than one expression that works.)

30)



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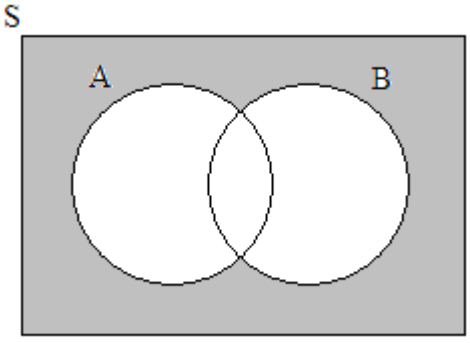

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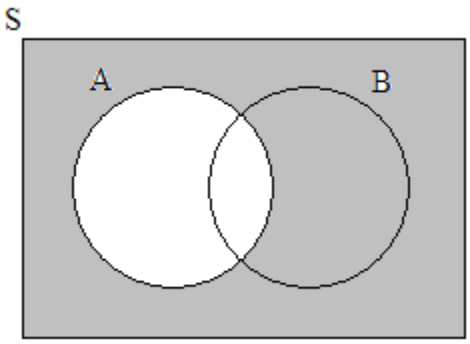

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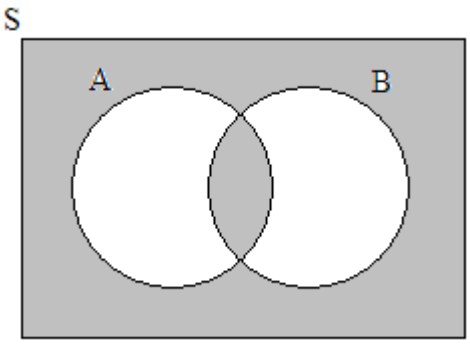

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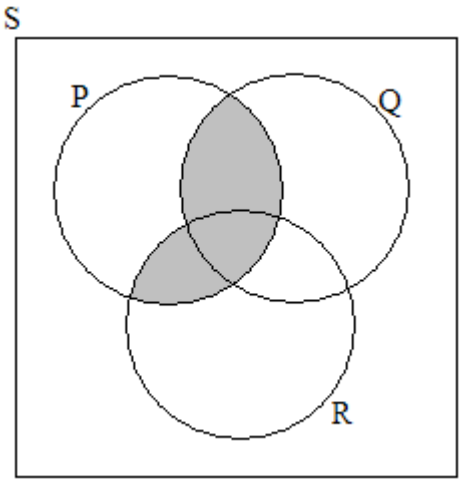

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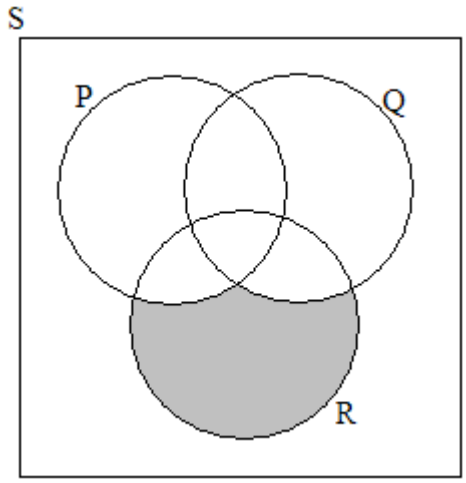
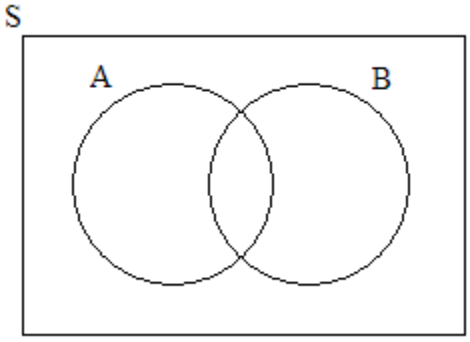
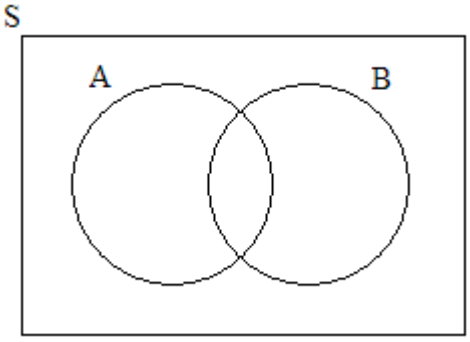


For the following problems, shade the areas that correspond with the expression given.

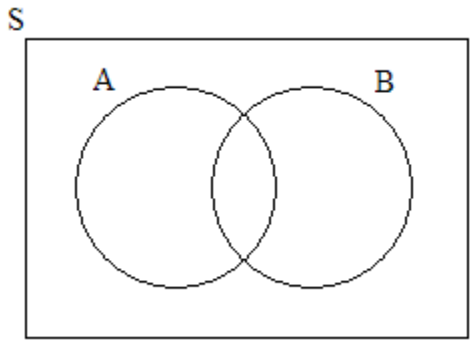
36) $S \cap A$



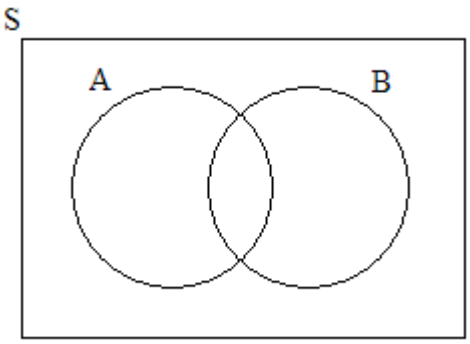
37) $A^C \cap B$



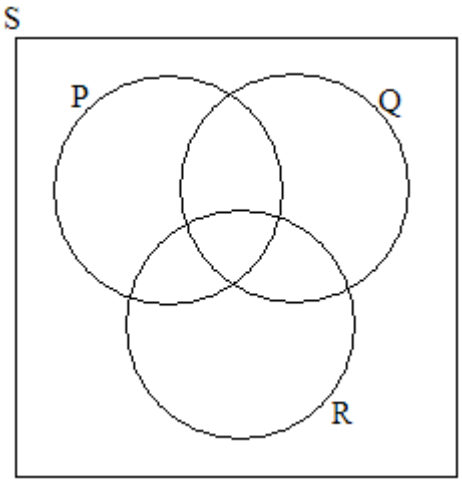
38) $(A \cap B) \cup A^C$



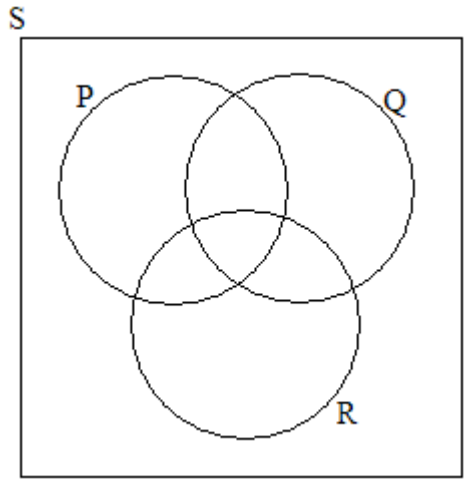
39) $(A \cup B)^C \cup (A \cap B)$



40) $P \cup (Q \cap R)^C$



41) $P^C \cap (Q \cup R)$



42) Suppose that School A has 48 teachers, and School B has 65 teachers. There are 15 teachers that teach at both School A and School B. Using the general counting formula, compute how many teachers are at School A and School B combined. Make a Venn Diagram and put the appropriate numbers in each section.

43) All the members of the Business Club are enrolled in an economics class, an accounting class, or both. 45 are enrolled in an economics class, 60 are enrolled in an accounting class, and 10 are enrolled in both. Using the general counting formula, compute how many students are in the club. Make a Venn Diagram and put the appropriate numbers in each section.

44) Over the last 50 days there was either rain or high winds. There were 35 days of rain, and 20 days of high winds. Using the general counting formula, how many days had both high winds and rain?

45) 45 patients at a hospital suffer from either fatigue or painful joints. 30 suffer fatigue, and 25 suffer from painful joints. Using the general counting formula, how many suffer from both painful joints and fatigue?

*For the following problems, use the Excel file **cars.xls**, and the commands COUNTIF or DCOUNT.*

Let J = {all Jeeps in the database}
Let F = {all Fords in the database}
Let Y = {all cars with a model year 1995 or later}
Let C = {all cars repaired in Colorado}

46) Find n(J)
47) Find n(F)
48) Find n($J \cap Y$)
49) Find n($J \cap Y \cap C$)
50) Find n($J \cup F \cup Y$)
51) Find n($F \cup Y \cup C$)
52) Use DeMorgan's Law to rewrite $(G \cap H)^C$, then draw a Venn Diagram which represents this.
53) Use DeMorgan's Law to rewrite $(G \cup H)^C$, then draw a Venn Diagram which represents this.
54) A school has 600 students. 200 are enrolled in a math class, 400 are enrolled in an English class, and 150 are in both a math and English class. How many are not enrolled in neither a math nor an English class?
55) A prison has 400 inmates. 250 are convicted of theft, 100 are convicted of drug trafficking, and 35 have been convicted of both crimes. How many have neither been convicted of theft nor drug trafficking?

Challenge and Application Problems

56) Read the Microsoft Word file ***United Way defends double counting.doc***. Let W = {dollars recorded by United Way in Waukesha County} and let M = {dollars recorded by United Way in Milwaukee County}. Answer the following questions:

a. Draw a Venn Diagram and fill in each section with the appropriate numbers.
b. Do you believe this practice is justified? Why or why not?

57) **Focus on the Project**: The following tasks are to be done on an individual basis and turned in with the Chapter Homework. This portion of the HW assignment should be typed up with all mathematical notation, equations, etc. properly formatted with the Equation Editor. Please save your file with an appropriate title so you can easily find it when you meet with your team to work on your team project. Before doing anything, read the Focus on the Project Section of this chapter.

Recall the variables:
Y = Years of experience.
T = Level of education.
C = Condition of the economy.
S = The event of a successful work out attempt.
F = The event of a failed work out attempt.

Find the following and save them somewhere safe for use later in the project. Make sure you label each one so you know what each one represents. Your answers should describe the result as a complete sentence. For example, for part [a], your answer might be something like, "The number of successful loan workouts that match our borrower's years of experience is 55."

a. $n(Y \cap S)$
(This is the number of Successful loan workouts that match your borrower's Years of experience, *Y*.)
b. $n(Y \cap F)$
c. $n(Y)$
d. $n(T \cap S)$
e. $n(T \cap F)$
f. $n(T)$
g. $n(C \cap S)$
h. $n(C \cap F)$
i. $n(C)$
j. $n(S_{BR})$ This is the number of successful workouts at BR bank
k. $n(F_{BR})$ This is the number of failed workouts at BR bank
l. $n(S_{CJ})$
m. $n(F_{CJ})$
n. $n(S_{DP})$
o. $n(F_{DP})$

p. Answer the question: What do these counts tell you about the likelihood of your particular client paying back the loan?

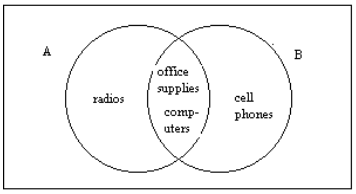
Save these counts on a spreadsheet for later use and label each answer for future reference.

Answers to Odd-Numbered Practice Problems

Chapter 2: Set Theory

1) True
3) True
5) True
7) False; $A \cap B = \varnothing$
9) True
11) False; $A \cap C = \{2, 4, 6\}$
13) {baseball, basketball, soccer, hockey, football, rugby}
15) {lacrosse}
17) All team sports except soccer, football, lacrosse, hockey, basketball, rugby, and baseball
19) {1, 2, 3, 4, 6, 8, 10}
21) $\varnothing$
23) {1, 3, 7, 9}

25)



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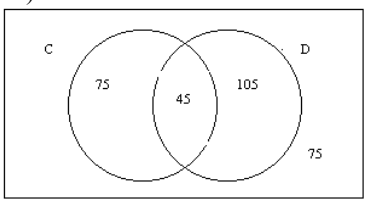

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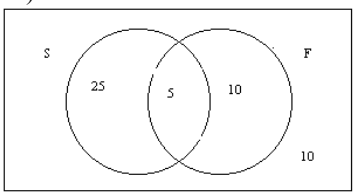
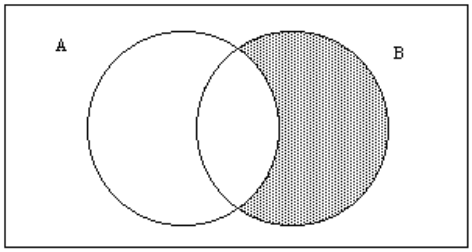


31) $(A \cup B)^C$ or $A^C \cap B^C$
33) $(A \cap B) \cup (A^C \cap B^C)$
35) $R \cap P^C \cap Q^C$

37)



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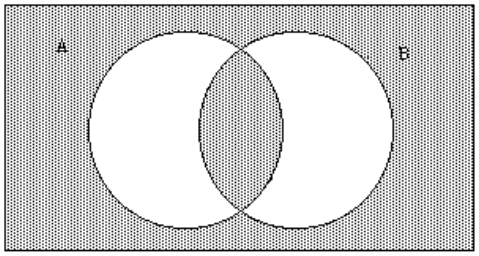

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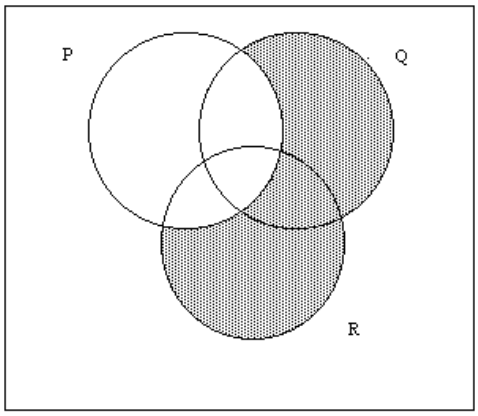

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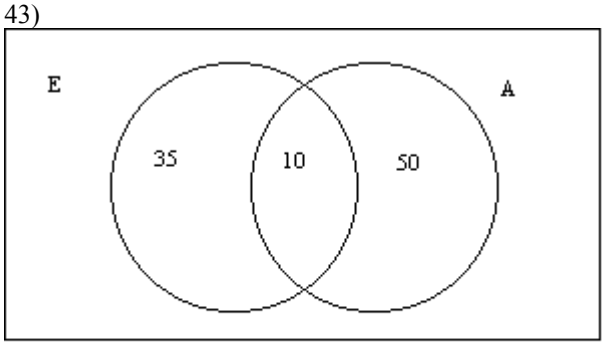


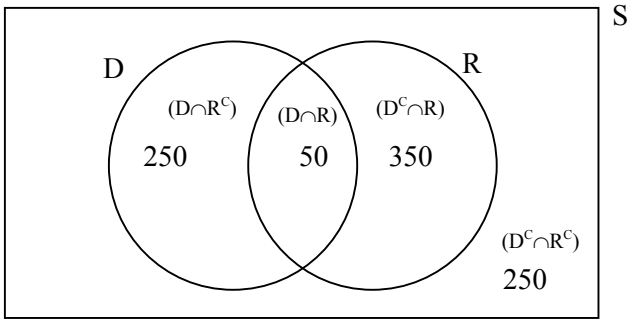
Answer: 95

45) 10
47) 94
49) 16
51) 306
53) $G^C \cap H^C$
55) 85

Chapter 3: Probability

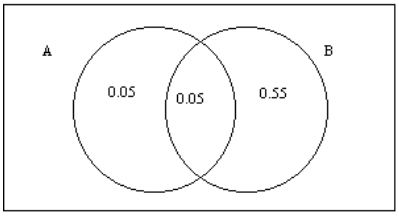
1) {HH, HT, TH, TT}
3) answers will vary. one possible answer: {any number less than 5,000}
5) 32.1%
7) 19.3%
9) 21.1%
11) 72.5%
13) 1/6
15) ½
17) 2/3
19) 2/3
21) 1/9
23) 7/12
25) ¾
27) 7/12
29) 31/36
31) disjoint
33) not disjoint

35) a)

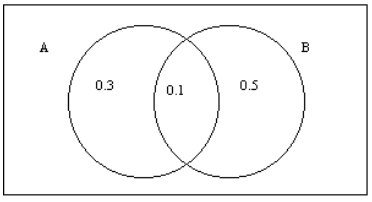


35 b) $P(D \cap R) = 50/900$ c) $P(D \cap R^c) = 350/900$ d) $P(R \cap D^c) = 350/900$ e) $P(R^c \cap D^c) = 250/900$
f) $P(D \cup R) = 650/900$ g) $P[(D \cap R^c) \cup (R \cap D^c)] = 600/900$

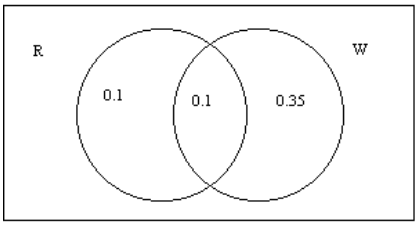
37) $P(A \cup B) = 0.65$



39) $P(A) = 0.4$



41) $P(R \cup W) = 0.55$



43) a) 38.9% b) 74.6%

Chapter 4: Expected Value

1) 30
3) 14
5) 66
7) –14
9) 170
11) $\sum_{i=1}^{10} x_i$
13) 1.638289573
15) 1898.279
17 a) $P(X \leq 4) = 0.89$ b) $P(X = 2) = 0.32$ c) $P(2 \leq X \leq 5) = 0.71$ d) $P(X > 5) = 0.04$
19) a) The probability that there are 2 or more cars at that household = 0.55
b) The probability that between 1 and 4 cars (including 1 and 4) are at that household = 0.89
c) The probability that 3 or less cars are at that household = 0.93
21) $0.42, player has advantage
23) 2.8 minutes
25) You lose $.1667 per game on the average so after 100 games you have last $16.67.
27) $3,250, proceed with the business venture

Chapter 5: Conditional Probability

1) $P(B|R)$
3a) the probability that a Sears battery is defective 3b) the probability that a defective battery was purchased at Sears
5a) $P(L|V) = 0.7$ 5b) $P(V|L) = 0.85$
7) 0.102 (rounded)
9) $P(H|B) = 0.357$ (rounded), you would need to know the total number of honor students at the school
11a) 88/195
11b) 88/1470
13) 1/9
15) No, because $P(E) = ½$, $P(10) = 3/36$, $P(E \cap 10) = 2/36$
17) 0.24
19) 0.012, or 1.2%
21) $P(U \cap D \mid A) = P(U|A) * P(D|A) = 0.22$

Chapter 6: Bayes' Theorem

1) no; it doesn't account for the entire universal set, there could be students taking both classes
3) yes. a student only has one GPA, which is in one of the categories mentioned
5a) 0.1603 b) 0.8397
7a) 0.81 b) 0.19 c) 0.004 d) 0.996
9a) 0.913 b) 0.223
11) P(C|D) = 0.364; P(B|D) = 0.546; P(A|D) = 0.091
13) a) 0.125 b) 0.5 c) 0.375 d) 0.333 e) 0.167 f) $P(U_3|R^C) = 0.5$

Chapter 9: Finance

1) $59.40
3) $58.87
5) $19.34
7) $2,186.43
9) $4,670.61
11) $1,742.43
13)$208,354.32
15) $1,809.83
17) 4.58 years
19) a) 17.14 years b) 16.85 years
21) a) 22.5 years b) 21.75 years
23) 3.03%
25) 4.59%
27) $538.33
29) $8558.37
31) $9,860.48
33) $11,905.05
35) $60.18
37) $19,903.87
39) $2,323.79
41) $123.14
43) $3,152.77
45) $36,333.76
47) $469.97
49) $11,224.30
51) $9,250.31
53) $13,343.52
55) $10,098.81
57) $37,804.33
59) $7,118.09
61) $9,199.45
63) $358.55
65) $587.44
67) $203,391.34
69) $1,675.48
71) $7,440.94
73) $539.47

Chapter 10: Project 3 Intro

1a) $2.25 b) $0
3a) –$5.05 b) –$7.30
5a) $50 b) $0
7a) $10 b) -$400

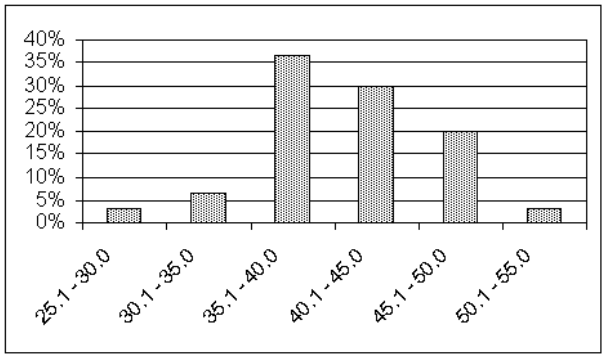
Chapter 11: Continuous Growth

1) 0.93514, 6.486% decrease
3) 1.03042
5) 1.00115
7) 1.00091
9) 0.03922
11) 0, because $e^0 = 1$
13) $5,620.60
15) $12,485.71
17) $2,095.55
19) 10.95 years
21) 8.51%
23) $39,455.37
25) 8 years, 8 months
27) $172,856

Chapter 12: Working With The Data

1) 28.7
3)

Bin	*Frequency*	*Relative Frequency*
30	1	0.033
35	2	0.067
40	11	0.367
45	9	0.300
50	6	0.200
55	1	0.033
More	0	
total	30	



5) min = 31.8, max = 47.5, average = 39.61, standard deviation = 3.88, variance = 15.06
7) mean = 6, range = 16, variance = 40, standard deviation = 6.324555
9) mean = $19,998.80, standard deviation = $1,323.45
11) 48.9%

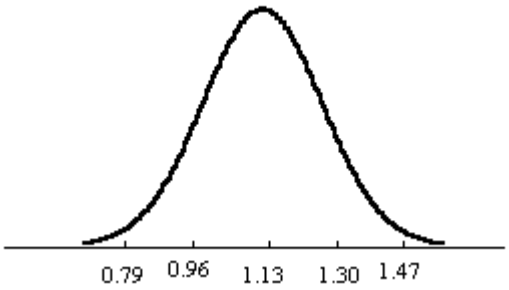
Chapter 13: Bootstrap Method

1-5) answers will vary
7) $2.00
9) $0, out of the money
11) $0, out of the money

Chapter 14: Black Sholes Method

1a) 0.98928 b) 0.15866
3 a) 298 days or more b) 234 and 298 days

5)



Index