

### Practice with Expected Value

1. You draw one card from a standard deck of playing cards. If you pick a heart, you will win \$10. If you pick a face card, which is not a heart, you win \$8. If you pick any other card, you lose \$6. Do you want to play? Explain.
2. The world famous gambler from Philadelphia, Señor Rick, proposes the following game of chance. You roll a fair die. If you roll a 1, then Señor Rick pays you \$25. If you roll a 2, Señor Rick pays you \$5. If you roll a 3, you win nothing. If you roll a 4 or a 5, you must pay Señor Rick \$10, and if you roll a 6, you must pay Señor Rick \$15. Is Señor Rick loco for proposing such a game? Explain.
3. You pay \$10 to play the following game of chance. There is a bag containing 12 balls, five are red, three are green and the rest are yellow. You are to draw one ball from the bag. You will win \$14 if you draw a red ball and you will win \$12 if you draw a yellow ball. How much do you expect to win or lose if you play this game 100 times?
4. A detective figures that he has a one in nine chance of recovering stolen property. His out-of-pocket expenses for the investigation are \$9,000. If he is paid his fee only if he recovers the stolen property, what should he charge clients in order to breakeven?
5. At Tucson Raceway Park, your horse, Soon-to-be-Glue, has a probability of  $\frac{1}{20}$  of coming in first place, a probability of  $\frac{1}{10}$  of coming in second place, and a probability of  $\frac{1}{4}$  of coming in third place. First place pays \$4,500 to the winner, second place \$3,500 and third place \$1,500. Is it worthwhile to enter the race if it costs \$1,000?
6. Your company plans to invest in a particular project. There is a 35% chance that you will lose \$30,000, a 40% chance that you will break even, and a 25% chance that you will make \$55,000. Based solely on this information, what should you do?
7. A manufacturer is considering the manufacture of a new and better mousetrap. She estimates the probability that the new mousetrap is successful is  $\frac{3}{4}$ . If it is successful it would generate profits of \$120,000. The development costs for the mousetrap are \$98,000. Should the manufacturer proceed with plans for the new mousetrap? Why or why not?
8. A grab bag contains 12 packages worth 80 cents apiece, 15 packages worth 40 cents apiece and 25 packages worth 30 cents apiece. Is it worthwhile to pay 50 cents for the privilege of picking one of the packages at random?

**Answers:**

1.

Let  $X$  be the random variable that takes on the values 10, 8 and  $-6$ , the values of the winnings. First, we calculate the following probabilities:

$$P(X = 10) = \frac{13}{52}, P(X = 8) = \frac{9}{52}, \text{ and } P(X = -6) = \frac{30}{52}.$$

The expected value of the game is

$$\begin{aligned} E(X) &= P(X = 10) * 10 + P(X = 8) * 8 - P(X = -6) * 6 \\ &= \frac{13}{52} * 10 + \frac{9}{52} * 8 - \frac{30}{52} * 6 \\ &= \frac{130 + 72 - 180}{52} \\ &= \frac{22}{52} \end{aligned}$$

Since the expected value of the game is approximately \$.42, it is to the player's advantage to play the game.

2.

This is very similar to the first problem. Let  $X$  be the random variable that takes on the values 25, 5, 0, -10, -15, the values of the winnings. A simple calculation yields the following probabilities:

$$P(X = 25) = \frac{1}{6}, P(X = 5) = \frac{1}{6}, P(X = 0) = \frac{1}{6}, P(X = -10) = \frac{2}{6}, \text{ and } P(X = -15) = \frac{1}{6}.$$

The expected value is given by

$$\begin{aligned} E(X) &= 25 * \frac{1}{6} + 5 * \frac{1}{6} + 0 * \frac{1}{6} - 10 * \frac{2}{6} - 15 * \frac{1}{6} \\ &= \frac{25 + 5 + 0 - 20 - 15}{6} \\ &= -\frac{5}{6} \end{aligned}$$

Therefore, Señor Rick is not loco since the expected value is approximately  $-.83$ .

3. Here, the gross winnings are 14, 12, or 0. Since you must pay \$10 to play, the net winnings are 4, 2, and -10. Let  $X$  be the random variable that takes on the values 4, 2, and -10, the values of the net winnings.

$$P(X = 4) = \frac{5}{12}, \quad P(X = 2) = \frac{4}{12}, \quad \text{and} \quad P(X = -10) = \frac{3}{12}.$$

The expected value of the game is given by

$$\begin{aligned} E(X) &= 4 * \frac{5}{12} + 2 * \frac{4}{12} - 10 * \frac{3}{12} \\ &= \frac{20 + 8 - 30}{12} \\ &= -\frac{2}{12} \end{aligned}$$

You should expect to lose \$16.67 after one-hundred games.

4. In this problem we want to determine the detective's fee so that the expected value is zero. Let  $y$  be the amount of his fee. Let  $X$  be the random variable that takes on the values  $y$  or 0, the amount he charges for a job. Then  $P(X = y) = \frac{1}{9}$  and  $P(X = 0) = \frac{8}{9}$ .

The detective is out \$9,000 regardless of whether he recovers the stolen property. So we have

$$\begin{aligned} E(X) &= (y - 9000) * P(X = y) - 9000 * P(X = 0) \\ &= (y - 9000) * \frac{1}{9} - 9000 * \frac{8}{9} \\ &= 0 \end{aligned}$$

Solving for  $y$  we see that the detective must charge \$81,000.

5. The gross winnings are \$4,500, \$3,500 and \$1,500. The net winnings are \$3,500, \$2,500, \$500 and -\$1,000. Let  $X$  be the random variable that takes on the value of the net winnings.

Then  $P(X = 3500) = \frac{1}{20}$ ,  $P(X = 2500) = \frac{1}{10}$ ,  $P(X = 500) = \frac{1}{4}$ , and  $P(X = -1000) = \frac{12}{20}$ .

$$\begin{aligned} E(X) &= 3500 * \frac{1}{20} + 2500 * \frac{2}{20} + 500 * \frac{5}{20} - 1000 * \frac{12}{20} \\ &= \frac{3500 + 5000 + 2500 - 12000}{20} \\ &= -50 \end{aligned}$$

So it would appear that Soon-to-be-Glue is soon to be glue.

6.

Let  $X$  be the random variable that takes on takes on the value of the investment.  
 $P(X = -30000) = 0.35$ .,  $P(X = 0) = 0.4$  and  $P(X = 55000) = 0.25$ .

The expected value of the project is

$$\begin{aligned} E(X) &= -30000 * 0.35 + 0 * 0.4 + 55000 * .25 \\ &= \$3,250 \end{aligned}$$

Since the expected value is positive, you should proceed with the project.