MAT116 Project 3
Chapter 11

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Types of Compounding

- Discrete
  - Interest is earned every month, week, day, etc.
  - We will explore this later
- Continuous
  - Interest is earned continuously
  - This is the type of compounding we'll use for this project

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Discrete Compounding

- For discrete compounding,
  - F = future value
  - P = present value
  - i = interest rate <u>per period</u>
  - n = total number of compounding periods
  - We will explore this formula more later

$$F = P(1+i)^n$$

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Continuous Compounding

- For continuous compounding,
  - F = future value
  - P = present value
  - r = annual interest rate
  - t = years.

$$F = Pe^{rt}$$

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Example

- Let an investment of $500 earn annual interest of 5.5% for 10 years.
- What is the future value of this investment? (The present value is $500)
- Recall that e^x in Excel is EXP(x)

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Work Space

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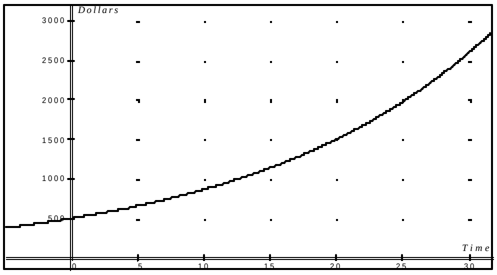
A Graph of Continuous Compounding

- Here is the investment over time.



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Example

- Suppose that a couple invests $2500 in an account that earns 4.3%, compounded continuously. How long before they earn $1000 in interest?

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Example

$$2500 + 1000 = 2500e^{.043t}$$

$$3500 = 2500e^{.043t}$$

$$\frac{3500}{2500} = e^{.043t}$$

$$1.4 = e^{.043t}$$

$1000 interest

Divide both sides by 2500

Now What?

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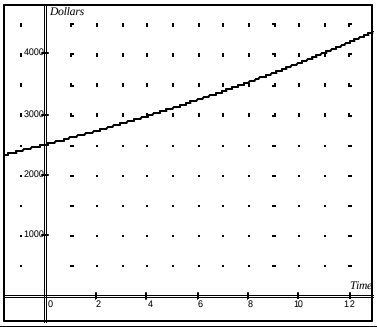
Let's Look at a Graph



Natural Logarithms

- Recall the meaning of $p = \log_b n$
  - This means $b^p = n$
- Example: $\log_{10} 1000 = 3$
  - because $10^3 = 1000$
- Example: $\log_2 16 = 4$
  - because $2^4 = 16$

Natural Logarithms

- When *e* is the base, then the logarithm is called the natural logarithm and is denoted by ln.
- Hence: $\log_e x = \ln x$

Example

- What is ln 1000?
- The result is NOT 3, since the base is not 10.
- We are looking for the power, p, that makes $e^p = 1000$.

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Example

- In order to more easily compute ln(1000), we can use a calculator or Excel.
- ln(1000) ≈ 6.908
  - This means $e^{(6.908)} \approx 1000$

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Back to Our Original Example

$$1.4 = e^{.043t}$$

$$\ln 1.4 = \ln\left(e^{0.043t}\right)$$ (Take ln of both sides)

$$\ln 1.4 = \ln e^{.043t}$$ (Remove ( )'s)

$$\ln 1.4 = 0.043t \ln e$$ (Move exponent. (Why?))

$$\ln 1.4 = 0.043t$$ (ln(e) = 1)

$$\frac{\ln 1.4}{0.043} = t$$ (Divide by .043)

$$7.825 \approx t$$

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Ratios

- With natural logs as a tool, we are ready to move forward in the project.
- We will eventually be interested in comparing our stock prices in one week to the one preceding it.
- We will do this by computing the ratio of the future value to the present value.

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Ratios

- The amount that a stock grows in one week, called the weekly ratio, can be expressed in terms of ratios as:

$$R = \frac{F}{P}$$

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Example

- A week ago, the stock of a company was $50.43. This week, it's value is $51.62. What is the weekly ratio and what does it mean?

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Ratios

- When the growth ratio is greater than one, we know the stock has increased in value.
- When the growth ratio is smaller than one, we know the stock had decreased in value.

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Example

- A week ago, the stock of a company was $50.43. This week, it's value is $48.21. What is the weekly ratio and what does it mean?

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Formulas for Ratios

- For continuous compounding:

$$F = Pe^{rt} \Rightarrow \frac{F}{P} = e^{rt}$$

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Formulas for Ratios

- For Discrete Compounding

$$F = P(1+i)^n \Rightarrow \frac{F}{P} = (1+i)^n$$

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Example

- An investment is growing at a monthly rate of 0.5%.
  - What is the monthly ratio?
  - What is the yearly ratio?
  - What is the annual yield?

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Example

- If a bank account compounds interest continuously at 10% (annually), what is the monthly ratio?
- F/P = 1.008368

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Focus on the Project

- The first step to pricing our stock option is to compute the weekly ratios from the data we have downloaded.
- This is easily done by dividing next week's adjusted closing price by the current closing price. Excel will do this easily for us.

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Focus on the Project

- We can use the fact that we know our Class Project risk-free rate is 4% (annually) to compute the weekly risk-free ratio. Keeping in mind that one week is 1/52 of a year:

$$
\begin{aligned}
R &= e^{rt} \\
&= e^{(.04)(1/52)} \\
&= 1.0007695
\end{aligned}
$$

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Focus on the Project

- The weekly risk-free ratio for our Class Project is 1.0007695.
- Note that I have kept several decimal places. You will want to do the same.

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Focus on the Project

- There is one last thing we can do at this point. Using the risk-free rate and the fact that we have a 20-week option period, we can get a preliminary estimate for the price of the stock price after this 20-week period.
- The total time needs to be converted to years:
  - t = 20/52 years

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Focus on the Project

- The closing price of DIS stock at the start of the option period was $21.87, so, our preliminary estimate for the stock value is:

$$
\begin{aligned}
F &= Pe^{rt} \\
&= 21.87e^{(.04)(21/52)} \\
&= 21.87(1.015503) \\
&= 22.21
\end{aligned}
$$

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