

### **Class Project: Client Info**

- Our Client
- Age Now: 35
- Retirement Age: 65
- Presumed Longevity: 90
- Retirement Goal: \$2,500,000

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### **Class Project: Client Info**

- Current Portfolio: \$50,000
- Money to Buy Options: \$10,000
- Current 401K Balance: \$40,000
- Monthly 401K \$: \$500
- Current Roth IRA Balance: \$12,000
- Current Yearly IRA \$: \$2,400

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### **Client Info: Stock Option**

- Company: Disney
- Strike Price: \$32.00
- Option Period: 20 weeks
- Risk Free Rate: 4%
- Cash to Buy Options: \$10,000
- Actual Closing Price: \$35.00

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### Class Project: Client Info

- Rate of Return (Pre-Retire): 7.5%
- Rate of Return (In-Retire): 5.5%
- Inflation Rate: 3.2%

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### Options Analysis

- Our Fair Price for DIS: \$1.85
- Total Available: \$10,000
- Options Purchased:  $10000/1.85 = 5405$
- Value of Option



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### Options Analysis

- Value of Option = \$3
- Profit per option =  $\$3 - \$1.85 = \$1.15$
- Total Profit =  $(\$1.15)(5405)$   
= \$6215.75
- New Portfolio Balance



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### Stock Portfolio Analysis: FV

- Current Amount: \$66,215.75
- What is the Future Value at the age of Retirement?

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### Stock Portfolio Analysis: FV

- We will use  $F = P(1+i)^n$
- What is  $i$ ?                      What is  $n$ ?



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### Stock Portfolio Analysis: FV

- Future Value =

$$F = 66215.75(1.00625)^{360}$$
$$= \$623,853.93$$

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### Stock Portfolio Analysis: PV

- What is the Present Value of this?
- Assume inflation acts yearly, not monthly.
- We will use the same formula, only in this form:

$$F = P(1+i)^n$$
$$P = \frac{F}{(1+i)^n}$$

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### Stock Portfolio Analysis: PV

- What is  $i$  and what is  $n$ ?

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### Stock Portfolio Analysis: PV

- We now compute the PV:

$$P = \frac{\$623,853.93}{(1+.032)^{30}}$$
$$= \frac{\$623,853.93}{2.5727}$$

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## Annuities

- An annuity is a financial object that provides payments (called rents,  $R$ ) at regular intervals over some period of time.
- An **ordinary annuity** makes payments at the end of each period.
- A **annuity due** makes payment at the start of each period.
- Annuities can be used to compute the amount of a loan payment (car, home, etc.) or the amount paid to someone from a retirement account.

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## FV of an Ordinary Annuity

- The formula for the FV of an ordinary annuity is:

$$F = R \left[ \frac{(1+i)^n - 1}{i} \right]$$

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## Example

- Suppose you invest \$1000 a year for 30 years and earn interest once a year at an annual rate of 5%. What is in the account at the end of the 30 years.

$$\begin{aligned} F &= R \left[ \frac{(1+i)^n - 1}{i} \right] \\ &= 1000 \left[ \frac{(1+.05)^{30} - 1}{.05} \right] \\ &\approx 1000 \left[ \frac{3.321942}{.05} \right] \approx 1000 [66.4388475] = \boxed{\phantom{000000}} \end{aligned}$$

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### Our Client: 401K FV

- Current Balance is \$40,000
- Monthly contribution: \$500
- $i = 0.00625, n = 360$
- We want to compute the FV of this account. To do this by hand, we will need to do TWO calculations. Why? What are they?

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### 401K FV: Part 1

- What is the FV of the client's series of payments?

$$F = R \left[ \frac{(1+i)^n - 1}{i} \right]$$
$$= 500 \left[ \frac{(1+0.00625)^n - 1}{.00625} \right]$$

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### 401K FV: Part 2

- What is the FV of the client's initial balance?

$$F = P(1+i)^n$$
$$= 40,000(1.00625)^{360}$$

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### 401K FV: Total

- Hence, the future value of the 401K account at the age of retirement is:

$$FV = \$673,722.71 + \$376,861.36$$



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### 401K FV: Excel

- To check our work in Excel, we use the FV command:
- =FV( *i*, *n*, *R*, Initial Balance)
  - Note: Excel calls the Initial Balance PV, for present value.

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### 401K in Today's Dollars

- What is our client's potential balance worth in today's dollars after taking into account inflation?
- Inflation rate = 3.2%

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## 401K in Today's Dollars

- Here's the calculation:

$$P = \frac{\$1,050,584 .07}{(1 + .032)^{30}}$$
$$= \frac{\$1,050,584 .07}{2.5727}$$



- For the purposes of our project, we will call this the "Present Value" of the 401K at retirement age after adjusting for inflation.

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## PV of an Annuity

- The text talks about the Present Value of an Ordinary Annuity from a different perspective.
- Recall that by saving \$500 a month for 30 years, the future value was \$673,722.71 .
- If we wanted to have \$673,722.71 in the account by depositing a lump sum today, what would that amount be? This amount is what the text calls the Present Value of the account.

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## PV of an Annuity

- Formula:

$$P = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$$

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## PV of an Annuity

- For our client

$$P = 500 \left[ \frac{1 - (1 + 0.00625)^{-360}}{0.00625} \right]$$
$$= 500 \left[ \frac{0.893860171}{0.00625} \right] = 500(143.01763)$$

- Instead of \$500 a month for 30 years, our client could deposit a lump sum of \$71,508.81.

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## PV of an Annuity

- Check:

$$F = P(1 + i)^n$$
$$= \$71,508.81 (1.00625)^{360}$$
$$= \$673,722.71$$

- We will assume, however, cannot come up with this lump sum, so will make monthly contributions as originally assumed

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## Our Client: Roth IRA FV

- The steps to compute the FV and the inflation-adjusted value of the Roth IRA is the **same** as for the 401K.
- Current Roth IRA Balance: \$12,000
- Current Yearly IRA \$: \$2,400 (\$200 per month)
- By Hand:
  - $\$113,058.41 + \$269,489.08 = \$382,547.49$
- In Excel:
  - $=FV(.00625, 360, 200, 12000) = \$382,547.49$

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## Our Client at Retirement

- Our client, then, has the following available at retirement:
  - Stock: \$623,853.93
  - 401K: \$1,050,584.07
  - Roth IRA: \$382,547.49
  - **TOTAL:** \$2,056,985.49
- Our client's goal was to have \$2,500,000, so that goal was NOT met.

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## What Is It Worth in Today's \$?

$$P = \frac{\$2,056,985.49}{(1+.032)^{30}}$$
$$= \frac{\$2,056,985.49}{2.5727}$$

- Our client's retirement account will be equal to this amount in today's dollars, after adjusting for inflation.

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## How Much Income?

- Our client has \$2,056,985.49 in future dollars at income.
- If we annuitize this amount over the course of his/her expected lifespan...
- What monthly and annual income will he/she receive?
- Now we assume a new rate of return,  $r = 5.5\%$
- We also have a new  $t = (90-65) = 25$  years.

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### How Much Income?

- Our post-retirement investments will compound monthly, so we have a new  $i$  and  $n$ .

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### How Much Income?

- We can use the Ordinary Annuity PV Formula to compute  $R$ , the monthly payment:

$$R = \frac{Pi}{1 - (1+i)^{-n}}$$
$$= \frac{(\$2,056,985.49)(0.00458333)}{0.74636487}$$

- This yields \$151,580.29 in annual income.

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### What's It Really Worth?

- After we factor in the effects of inflation, what is this monthly/annual income worth in today's dollars?

$$P = \frac{\$151,580.29}{(1 + .032)^{25}}$$
$$= \frac{\$151,580.29}{2.5727}$$

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### What's It Worth Over Time?

- Our client can expect the equivalent of \$58,918.52 per year (or \$4909.88 per month) in his/her first year/month of retirement.
- Keep in mind that inflation will continue to eat into your client's spending power.

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### What's It Worth Over Time?

- For example, 10 years into retirement, that annual income is worth the following (in today's dollars):

$$P = \frac{\$151,580.29}{(1 + .032)^{35}}$$

$$= \frac{\$151,580.29}{3.011545}$$

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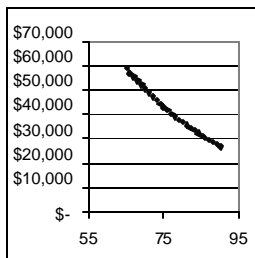
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### What's It Worth Over Time?

- Here's a graph of the value (in today's \$) of your client's annual income over time:




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### Advice

- Our client's goal at retirement was \$2,500,000, so he/she will be \$2,500,000 - \$2,056,985.49 = \$443,014.51 short.
- How much money should our client invest monthly at 7.5% (the assumed growth rate) to make up this difference?
- Basically, we need to the monthly payment needed to make up this difference.
- Hence, we need to know R given F

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### Advice

$$F = R \left[ \frac{(1+i)^n - 1}{i} \right]$$
$$\Rightarrow$$
$$R = \frac{Fi}{(1+i)^n - 1}$$

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### Advice

$$R = \frac{Fi}{(1+i)^n - 1}$$
$$= \frac{(\$443,014.51)(.00625)}{1347.45}$$
$$= \$328.78$$

- Hence, if our client saves about \$330 more per month, if that's possible, he/she can reach the stated retirement goal of \$2,500,000.

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