MAT116 Project 1 Chapter 5: Conditional Probability

5-1: Conditional Probability

- We need conditional probability when the probability of an event is dependent upon other events also happening (or not happening) or when we want to consider the impact of other events on our probability.
- Example: What is the probability that a loan workout will be successful *given that* a borrower has a college education.
- The probability here is thought to depend on education level. It's *conditioned* on another factor.

Notation

- $P(A|B)$ is read "The probability of *A*, given *B*," or "the probability of *A*, if *B*," which means the probability of event *A* occurring, if you know for certain that event *B* has occurred or will occur.

Example

- Example: Let R be the event that it rains and let W be the event that it is "windy."
  - P(R|W) is the probability that it rains given that it is also windy.
  - P(W|R) is the probability that it will be windy given that it is raining.
- P(R|W) is not necessarily the same or equal to P(W|R). In general…
- P(A|B) is not necessarily the same or equal to P(B|A).

The Formulas

- Two ways to compute conditional probabilities:

$$P(A|B) = \frac{n(A \cap B)}{n(B)}$$

Use this when you know the number of elements in the sets.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Use this when you know the probability of the events.

Example

- There is a 50% probability of rain (R) and a 10% probability of both rain and wind (W). What is the probability of wind given that it rains?
- Start with writing out everything in probability notation:
  - P(R) = 0.5   P(R∩W) = 0.1
  - P(W|R) = ?
  - What about P(R|W)?

Example

- There is a 30% probability of rain (R) and a 40% probability of wind (W). The probability of either rain or wind is 0.46. What is the probability of rain given that there is wind? (Ans: 0.6)

- Caution: This problem requires an extra step…what is it?

Example

- You have invested in Mini Co. stock because you think it will be acquired by Gigantic Conglomerate, Inc. (GCI) There is a 90% probability that GCI will acquire Mini if Mini shows a profit next month and there is an 80% probability that Mini will actually show that profit. What is the probability that Mini will show a profit and be acquired by GCI?

Solution

- A: GCI will acquire Mini
- B: Mini will show a profit
- Then we know: $P(A|B) = 0.9$ and $P(B) = 0.8$
- What is $P(A \cap B)$?
- Rearrange the following equation:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

5-2 & 5-3: Independence

- Definition: Two events are called **independent** if and only if the outcome of one event does not affect the outcome of the other.

Common Sense Independence

- Events *A* and *B* are independent if and only if P(A|B)=P(A) or P(B|A)=P(B).
- Why do I say this is "common sense?"

Mathematical Independence

- If events *A* and *B* are independent, then

$$P(A \cap B) = P(A) \cdot P(B)$$

- Likewise, if this equation is true, then *A* and *B* are independent
- Where does this come from?

Example

- In the US Population, the probability of being “Hispanic” is 0.11, the probability of living in CA is 0.12, and the probability of being a “Hispanic” living in CA is 0.04. Are the two events (being “Hispanic” and living in CA) independent?
- What is your initial guess?

Solution

- H: a person is “Hispanic”
- C: a person lives in CA
- $P(H) = 0.11$
- $P(C) = 0.12$
- $P(H \cap C) = 0.04$

Example

- According to the weather service, the probability of rain in NY is 0.5 and the probability of rain in Honolulu is 0.3. If the probability of rain in either of the two cities is 0.65, are the two events independent.
- What is your initial guess?

Solution

- N: Rain in New York
- H: Rain in Honolulu
- P(N) = 0.5
- P(H) = 0.3
- $P(N \cup H) = 0.65$
- Two (related) ways to check independence:
  - Is $P(N \cap H) = P(N)\ P(H)$?
  - Is $P(N|H) = P(N)$ or is $P(H|N) = P(H)$?

5-4: Extending the Idea

- If we have more than two events to pay attention to (such as years of experience, education level, state of the economy), we will want a way to test for independence and to compute conditional probabilities.

Extending the Formula

- If events A and B are independent and Z is another event (not necessarily independent of A and B), then:
  - $P(A \cap B\ |Z) = P(A|Z) \times P(B|Z)$
- If events A, B, and C are independent and Z is another event (not necessarily independent of A,B, and C), then:
  - $P(A \cap B \cap C\ |Z) = P(A|Z) \times P(B|Z) \times P(C|Z)$

Interpreting It

- P(A∩B |Z)…What does this mean?
- "The probability that both events A and B happen given that event Z happens."
- How does this relate to the project?

Focus on the Project

- Let S = event of a successful workout
- Let F = event of a failed workout
- Let Y = event 7 years of experience (our loan)
- Let T = event of bachelor's degree
- Let C = event of a normal economy
- f = full value of the loan
- d = default value of the loan
- r = foreclosure amount

Focus on the Project

- These are all relevant probabilities:
- P(S|T) P(S|Y) P(S|C)
- P(F|T) P(F|Y) P(F|C)
- How do we compute or estimate these?

Focus on the Project

- To estimate P(S|Y) we need to look at records from the BR bank since that is the only bank that has information on years of experience.
- Therefore: $P(S|Y) \approx P(S_{BR}|Y_{BR})$
- Using the formula, we have:

$$P(S \mid Y) \approx P(S_{BR} \mid Y_{BR}) = \frac{P(S_{BR} \cap Y_{BR})}{P(Y_{BR})} = \frac{n(S_{BR} \cap Y_{BR})}{n(Y_{BR})}$$

Focus on the Project

- Let's Do This in Excel!!!
- We should get $P(S|Y) \approx P(S_{BR}|Y_{BR}) = 0.4393$

Focus on the Project

- All the rest of the conditional probabilities listed below can likewise be estimated using our Excel data file.
  - P(S|T) P(S|Y) P(S|C)
  - P(F|T) P(F|Y) P(F|C)

Focus on the Project

- We can now find the expected value of a loan workout for a loan with 7 years of experience.
- That is, what is $E(Z_Y)$? Z_Y is the random variable that gives the amount of money returned from a workout attempt on a long with Y years of experience.
  - $E(Z_Y) = f \times P(S|Y) + d \times P(F|Y)$
- We can also compute $E(Z_C)$ and $E(Z_T)$

Focus on the Project

- We can also find the more complex probabilities:
  - $P(Y \cap T \cap C|S)$ and $P(Y \cap T \cap C|F)$
- Since Y, T and S were said to be independent then
  - $P(Y \cap T \cap C|S) = P(Y|S) \times P(T|S) \times P(C|S)$
  - $P(Y \cap T \cap C|F) = P(Y|F) \times P(T|F) \times P(C|F)$
- These are not the probabilities we ultimately want, of course, so more work is needed.

Project Progress

- Make sure you use your own team's data (not the class data) to do your computations.
- Make sure you record your results and save them.
- Have more than one person do them to make sure you all get matching results.
- Add a section to you written report that describes what you have done and how it affects (or does not affect) your decision to now.
- Please submit your written report in printed form for review by the instructor.