MAT116 Project 1
Chapter 4: Expected Value

4-1: Summation Notation

- Suppose you want to add up a bunch of probabilities for events E_1, E_2, E_3, … E_{100}.
- One way to write it would be:

$$P(E_1) + P(E_2) + P(E_3) + ... + P(E_{99}) + P(E_{100})$$

Summation Notation

- Another, more compact way to write it would be by using summation notation. This same set of probabilities, added together, can be expressed as follows:

$$\sum_{i=1}^{100} P(E_i)$$

Summation symbol

i is called the index

Summation Notation

- To add the first 25 whole numbers up, 1+2+3+…+24+25, we would write this:

$$\sum_{i=1}^{25} i$$

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Summation Notation

- To add the following sum: $3^2+4^2+\ldots 24^2+25^2$

$$\sum_{i=3}^{25} i^2$$

Note that the index starts at 3 instead of 1 to match the sequence of numbers you are adding

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Example

- Find the value of the following:

$$\sum_{i=3}^{6} (5+2i)$$

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Optional Examples

- Find the value of the following:

$$\sum_{j=0}^{6}(-1)^{j} \quad \sum_{k=3}^{7}(k-1) \quad \sum_{i=1}^{5}2i^{-1}$$

Optional Examples

- Write the following in summation notation:
  - $3(2)^2 + 3(3)^2 + 3(4)^2 + \ldots + 3(28)^2 + 3(29)^2$
  - $F(0.5)+F(1.5)+F(2.5)+F(3.5)+\ldots+F(10.5)$
  - $3(2)^2 - 3(3)^2 + 3(4)^2 - 3(5)^2 \ldots + 3(20)^2 - 3(21)^2$

4-2: Sums and Probability

- Summation Notation will come in handy when we want to add the probabilities of several events at once.

4-3: Excel and Sums

- Find the value of the following using Excel
- Note how hard this would be to write our or compute by hand!

$$\sum_{i=7}^{25}\left(2i^2 - 10\right)$$

4-4: Random Variables

- A random variable is a variable whose value can change. In the context of probability, it is usually the numerical outcome of some random trial or experiment.
- For example, throwing a die has an associated random variable. Let V be the number that comes up on the die. The outcome, and one of the members of {1,2,3,4,5,6} is random and so V is a random variable.

Notation

- Suppose V is the random variable just described for throwing a die. We will often denote probabilities as follows:
  - P(V=1) = 1/6

This the probability that the die comes up as a 1

Notation

- $P(2 < V = 5) = ???$
- This is the probability that the number that comes up on the die is greater than 2 and less than or equal to 5.
- So, what is $P(2 < V = 5)$

Example

- Let T be the random variable that gives the total of rolling two dice.

- What is $P(T > 7)$?
- What is $P(4 < T = 10)$?

4-5: Expected Value

- The Expected Value of a Random Variable is the predicted average of all outcomes of a very large number of trials or random experiments.
- It is the value you expect to get (as an average) and may not actually be equal to any of the outcomes that are possible in your experiment.
  - For example, if there are 100 slips of paper in a hat (50 with 1 written on them and 50 with 0 written on them), what is the average value of a slip you pull out of the hat if you pull out “enough” slips of paper?

Expected Value

- Suppose you have 60 plastic markers in a box. 20 are marked with as \$3, 20 are marked as \$4, and 20 are marked as \$5.
- If you randomly choose one of the markers out of the bag many many times, what is the average (expected value) of such an action? How can you find the answer without doing any computations?

Example

- Now change the problem so it reads like this? Suppose you have 60 plastic markers in a box. 20 are marked with as \$3, 10 are marked as \$4, and 30 are marked as \$5.
- Do you think the expected value will be the same as before? Smaller? Larger? Why?
- HOW WOULD YOU FIND SUCH A VALUE?

Definition of Expected Value

- If X is a random variable, then $E(X)$, , and μ_X can all represent the expected value of X
- If there are n different numerical outcomes of a trial, the formula for Expected Value is:

$$E(X) = \sum xp = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$

- where x is each possible value of the random variable, and p is the probability of each outcome occurring.

What does this mean?

$$E(X) = \sum xp = x_1 p_1 + x_2 p_2 + ... + x_n p_n$$

- Note that each value of the random variable gets multiplied by its corresponding probability.
- So, if a the probability of a particular outcome is large, then it gets multiplied by a larger value. Hence, it will play a larger role in the final expected value result. We say that it i s weighted more heavily.
- Likewise, an outcome with only a small probability of happening gets multiplied by a much smaller value and so it is weighted much less.

Back to our Example

- Now change the problem so it reads like this? Suppose you have 60 plastic markers in a box. 20 are marked with as $3, 10 are marked as $4, and 30 are marked as $5.
- Start by building a probability table that includes columns for the random variable, its corresponding probability, and the product of the two. Each row of the table will correspond to a single outcome of the random variable.

Continuing our Example

x	p	x*p
$3		
$4		
$5		
	Expected Value=	

Example

- Let *S* be the sample space represented by all possible outcomes of tossing three coins on a table.
- Let X = the number of heads that occur in a trial (of tossing the three coins).
- What is the expected value of X?

Group Activity (Time allowing)

- Let *S* be the sample space represented by all possible outcomes of tossing four coins on a table.
- Let X = the number of heads that occur in a trial (of tossing the four coins).
- What is the expected value of X?

Example

- A 27-year old woman decides to pay $156 for a one-year life -insurance policy with coverage of $100,000. The probability of her living through the year is 0.9995 (based on data from the US Dept of Health and AFT Group Life Insurance). What is her expected value for the insurance policy. (Ans: -$106)

Example

- When you give a casino $5 bet on the number 7 in roulette, you have a 1/38 probability of winning $175 (including your $5 bet) and 37/38 probability of losing $5. What is your expected value? In the long run, how much will you lose for each dollar bet?
- (Ans: E(X) = -$0.26316)

Example

- **A carton has 20 batteries in it. 2 are defective. You randomly choose three from the carton. If X is the number of defective batteries pulled in the random pick of 3, then P(X=0) = 0.716, P(X=1)=0.268 and P(X=2) = 0.016. (These numbers come from more complicated formulas we have not covered in this class.)**
- **Explain what each of these mean in your own words**
- **Compute the Expected Value of X and explain what it means in words.**

Example

- Suppose you insure a $500 iPod from defects by paying $60 for two years of coverage. If the probability of the unit becoming defective in that two-year period is 0.1, what is the expected value of that insurance policy?
- Ans: -$10

Focus on the Project

- Let *S* be the event that an attempted loan workout is successful
- Let *F* be the event that an attempted loan workout fails
- Let *Z* be the random variable that gives the amount of money that Acadia Bank receives from a future loan workout.

Focus on the Project

- We can use the probability of failure and success to find a preliminary estimate for the expected value of Z
- Recall that P(S) = 0.464 and P(F) = 0.536
- Doing these computations gives E(Z) ≈ $1,990,000
- This is lower than the foreclosure amount of $2.1 million, so at this point we would conclude that foreclosing is a better way to go.