MAT116 Project 1 Chapter 3: Probability

Section 3-1: Introduction

- Probability will play the central role in solving Project 1. Ultimately, our decision to work out or foreclose depends on the probability of success or failure on the workout. This will determine how much money we will recover from the loan.

Definitions

- A **Trial** is an activity where the result is unknown. (this is **sometimes** called an **experiment**, or a **random experiment**)
- An **Event** is a result of a trial. This is usually represented by a capital letter.
- An **Outcome** is one specific result of a trial.
- A **Sample Space** is the set of all possible outcomes. It is usually represented by a capital *S*, but we will use the symbol *§* to represent sample space. (*S* is needed for something else)
- A **Probability** is the proportion of times a particular event will occur. It is usually represented by a capital *P*.

Example

- You have three nickels (coins) that you flip into the air and onto the table.
- A **trial** would be flipping the three coins.
- An **event** is one tail and two heads, but note that there may be more than one way (events) that give you this: HTH, HHT, THH
- One possible **outcome** would be heads-tails-heads (HTH for short).

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Example

- The **sample space** for this trial is all the possible outcomes. (8 in this case.)
  - HHH HHT
  - HTH HTT
  - TTT TTH
  - THT THH

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Some Notation

- Suppose we are interested in knowing all the possible outcomes where we get two heads and one tail. We will call this event A:
- We can write A in set notation:
  - A = {HHT, HTH, THH}
- There are therefore 3 ways this can happen out of a total of 8 possible outcomes. We say the probability of the event A happening is 3/8.

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Theoretical Probability

- Let *E* be an event. Then:
  - *P(E) = k/n*
    - *k* = the number of ways event *E* can occur
    - *n* = the total number of possible outcomes

$$P(E) = \frac{k}{n}$$

- *CAUTION: This formula is only valid if each outcome is equally likely.

Example

- A marble is drawn from a bag. There are 15 red, 12 yellow, and 18 blue marbles in the bag.
- What is the probability of randomly drawing a single red marble from the bag?
- What is the probability of randomly drawing a single blue marble from the bag?

Question for Discussion

- A marble is drawn from a bag. There are 15 red, 12 yellow, and 18 blue marbles in the bag.
- What is the probability of drawing a red marble from the bag, setting it aside, and then immediately drawing a second red marble?
- Does the result change if you replace the first red marble before drawing the second time? If so, why?

Empirical Probability

- Empirical data is that which you observe. For example, you have collected data that indicates that of the last 550 loans a bank granted, 42 of them were foreclosed upon.
- Then based on the empirical data, you might say that the probability of a loan going into foreclosure is 42/550.

Empirical Probability

- Let *E* be an event. Then:
- *P(E) = k/n*
  - *k* = the number of times event *E* has occurred **in the past** (under similar circumstances)
  - *n* = the number of trials **in the past**

$$P(E) = \frac{k}{n}$$

Law of Large Numbers

- Empirical probabilities are basically just estimates. They do not necessarily predict the outcome of a particular trial. ("The next loan will go to foreclosure!") We do know this, however: **The outcome of one trial cannot be predicted, but, one can predict what will happen over a series of many trials.**

3-2: Combining Events

- A marble is drawn from a bag. There are 15 red, 12 yellow, and 18 blue marbles in the bag.
- What is the probability of randomly drawing either a yellow or blue marble from the bag?
- How could we right this using notation from our Set Theory Chapter?
- What about: “What is the probability of not picking a red marble?”

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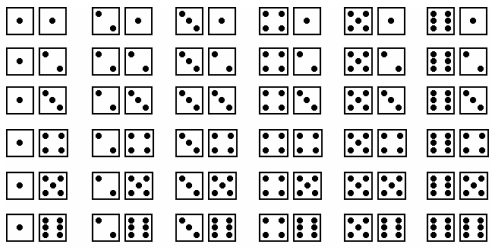
3-3: Dice Problems

- Suppose you have two dice that you roll onto a table. Here is the sample space.



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Examples

- What is the probability of getting a total of 10?
- What is the probability of getting the same number on each die?
- What is the probability of getting a prime number total?
- What is the probability of NOT getting a total of 7?
- What is the probability of getting either a total of 8 or a total of 11?
- What is the probability of getting either a 4 or an odd number on one of the dies?
- What is the probabilit of getting a 4 and an odd number on the other die?

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3-4: Set Theory and Probability

- The idea of randomly choosing one element of a set can be applied to probability. Remember that a sample space, $\S$, is a set of all possible outcomes. Think of the event E as the *set* of all the outcomes in which this event occurs. E would be a subset of $\S$.
- For example, when rolling one die, $\S = \{1,2,3,4,5,6\}$. Let the event E = rolling an even number. Then the *set* E is all the ways to roll an even number: $E = \{2,4,6\}$. This is a subset of $\S$.

3-5: Rules of Probability

- Let A and B be events, and let $\S$ be the sample space.
- **Rule 1:** $0 \le P(A) \le 1$
- **Rule 2:** $P(\S) = 1$
- **Rule 3:** $P(A^C) = 1 - P(A)$
- **Rule 4:** If A and B are disjoint, then
  - $P(A \cup B) = P(A) + P(B)$

3-6: Venn Diagrams

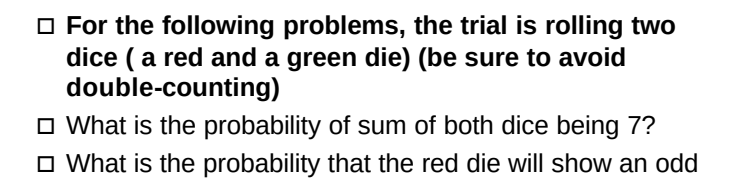
- A total of 70 students are randomly interviewed. 23 own a car. 45 own a bike. 18 own both a car and a bike. Draw a Venn diagram that displays all of the probabilities related to this survey.

3-6B General Probability Formula

- If *A* and *B* are events, then
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Subtracting compensates for the double-counting error.

Examples

- **For the following problems, the trial is rolling two dice (a red and a green die) (be sure to avoid double-counting)**
- What is the probability of sum of both dice being 7?
- What is the probability that the red die will show an odd number or the sum of the two dice will be 8?
- What is the probability that the green die is 6 or the sum of the two dice is 10?
- What is the probability that the green die shows an even number *and* the sum of the two dice is 10?



Group Exercise

- See #36 from Chapter 3
- From a survey involving 1,000 people in a certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. (Barnett p. 414)

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Group Exercise

- Draw and label a Venn Diagram that demonstrates this information. Include the sample space as part of you diagram.
- Give or compute the probability that a randomly selected person from the city has tried both of the colas. Write your answer in probability notation. Example: P(D) = 0.356
- Give or compute the probability that a randomly selected person from the city has tried the diet cola but not the regular cola. Write your answer in probability notation.
- Give or compute the probability that a randomly selected person from the city has tried the regular cola but not the diet cola. Write your answer in probability notation.

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Group Exercise

- Give or compute the probability that a randomly selected person from the city has tried neither of the colas. Write your answer in probability notation.
- Give or compute the probability that a randomly selected person from the city has tried either the diet or the regular cola. Write your answer in probability notation. Try to see if you can compute this in TWO different ways, one which uses DeMorgan's Law and one which uses the General
- Give or compute the probability that a randomly selected person from the city has tried one of the colas but not both. Write your answer in probability notation. Be careful on this one…a picture should help.

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Focus on the Project

- Let *S* be the event that an attempted work out is successful and let *F* be the event that it fails. Use the **COUNTIF** function to find the fraction of past work outs which were successful. This fraction is our estimate for *P*(*S*). Likewise, we find the fraction of attempts that failed and use this as our estimate for *P*(*F*).

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Focus on the Project

- **Example 1:**
- Use the ***Loan Records.xls*** to estimate *P*(*S*).
  - DCOUNT will work
  - COUNTIF will also do the job

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