

Business Mathematics I¹

Sample Midterm Examination 1 Solutions

1. (i) $P(I \text{ and } H) = P(I \cap H)$. $P(I \cap H) = P(I) + P(H) - P(I \cup H)$
 $= 0.6 + 0.3 - 0.7 = 0.2$

(ii) $P(I^C \cap H^C)$ is the probability that both stocks go down tomorrow.

$$P(I^C \cap H^C) = P((I \cup H)^C) = 1 - P(I \cup H) = 1 - 0.7 = 0.3$$

(iii) Out of a huge number of days with similar conditions, IBM stock would go up or stay the same following approximately 60% of the days.

2. (i) $P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.2 + 0.5 + 0.2 = 0.9$

(ii) $E(X) = 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + 3 \cdot P(X = 3) + 10 \cdot P(X = 10)$
 $= 1 \cdot 0.2 + 2 \cdot 0.5 + 3 \cdot 0.2 + 10 \cdot 0.1$
 $= 2.8$

(iii) $\sum_{i=1}^2 P(X = i) = P(X = 1) + P(X = 2) = 0.2 + 0.5 = 0.7$

3. (i) Let $S = \{\$1\$1, \$1\$5, \$5\$1, \$5\$5\}$, and define the probability of each of the four outcomes to be 0.25.

(ii) X can assume the values of \$2, \$6, or \$10.

$$\begin{aligned} E(X) &= \$2 \cdot P(X = \$2) + \$6 \cdot P(X = \$6) + \$10 \cdot P(X = \$10) \\ &= \$2 \cdot 0.25 + \$6 \cdot 0.50 + \$10 \cdot 0.25 \\ &= \$6 \end{aligned}$$

(iii) If we repeated the experiment a huge number of times, the average of the total values of the two bills selected would be approximately \$6.

4. (i) We are looking for $P(E|J)$. $P(E|J) = \frac{P(E \cap J)}{P(J)} = \frac{0.1}{0.2} = 0.5$

(ii) $P(E|J) = 0.5$ and $P(E) = 0.3$. Since these are different, E and J are **not** independent.

5. We are looking for the probability of the event $C \cap W^C$. Since C and W are independent, so are C and W^C .

$$P(C \cap W^C) = P(C) \cdot P(W^C) = P(C) \cdot (1 - P(W)) = 0.6 \cdot (1 - 0.7) = 0.18$$

6. The events R and S partition a natural sample space for the experiment.

$$P(F) = P(F | R) \cdot P(R) + P(F | S) \cdot P(S) = 0.8 \cdot 0.3 + 0.4 \cdot 0.7 = 0.52$$

7. (i) $P(S) \cong 4/10$

$$(ii) P(S | H) = \frac{P(S \cap H)}{P(H)} \cong \frac{1/10}{3/10} = \frac{1}{3}$$